Perspective invariant movie analysis for depth recovery

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ABSTRACT

Processing entire movies and taking advantage of the inter-frame redundancy is the key of shape-from-motion analysis. Thus, recovering the depth of a fixed scene from an image sequence can be viewed as a movie processing problem : how to focus the redundant depth information of a noisy image sequence into a perfect depth-coherent movie ? We present a natural set of axioms in agreement with the depth recovery, in the simple case of a straight movement of the camera parallel to the focal plane. According to these axioms, we show that there is a unique depth-coherent way of processing movies, described by a nonlinear partial differential equation. The corresponding multiscale analysis has the property of smoothing the motion field of a movie, leading naturally to a perfect motion field compatible with a depth interpretation. Moreover, in the case of an ideal movie, i.e. coherent with the observation of a fixed 3D scene, this analysis can be viewed as a simple filtering of the camera movement preserving the depth interpretation given by the movie, and is thereby perspective invariant. Last, we study a numerical scheme, compatible with the theoritical axioms, and produce some experiments on synthetic noisy movies.

key-words : shape from motion, image sequence processing, perspective invariance, multiscale analysis.

1 INTRODUCTION

One fundamental problem in robotics is the reconstruction of a geometric environment from one or several observations. This depth recovery problem has many applications, like navigation, cartography, and shape analysis. Among explorated techniques in image processing, often referenced as "shape from X", where X is one of stereovision, shading, texture, etc ..., we focused our attention on shape from motion. By analyzing a sequence of images, we can hope to recover depth more accurately than other techniques do, according to general properties of active vision[1], which states that the redundancy resulting from camera motin should naturally bring robustness and accuracy. In a way, the analysis of image sequences (20 or more frames) with known camera movement is a generalization of stereovision (2 images).

In order to take advantage of this redundancy, we need to analyze movies globally. Thus, the depth recovery problem can be reformulated in terms of movie processing : how to filter a noisy and depth-incoherent movie, from which a direct scene reconstruction is almost impossible, into a perfect, depth-coherent movie ? We give an answer in the case of a simple camera movement, a translation parallel to the focal plane. After introducing the general framework, we state by an axiomatic approach the existence and unicity of such a movie analysis. Then, we study its properties in terms of image and scene interpretation, showing that it is perspective invariant in a certain sense. Last, we present a numerical scheme to apply this process to digitized image sequences, and show some conclusive experiments on noisy synthetic movies.

2 GEOMETRIC FRAMEWORK

Let $Z(X,Y)$ represent a surface in $\mathbb{R}^3$, observed by a unit focal length camera located at $(C,0,0)$, looking towards the Z axis (cf. figure 1). Then under perspective projection, a point $M(X,Y,Z(X,Y))$ of the surface is projected onto $P(x,y)$ in the image plane, with

$$x = \frac{X-C}{Z(X,Y)} \quad \text{and} \quad y = \frac{Y}{Z(X,Y)}. \tag{1}$$

We now suppose that the camera has a straight movement along the X axis, given by its position $C(\theta)$ where θ is the time variable. Thus, calling $U(X,Y)$ the perceived isotropic luminosity of $M(X,Y,Z(X,Y))$, we can state the fundamental relation between the image and the scene space,

$$u\left(\frac{X-C(\theta)}{Z(X,Y)}, \frac{Y}{Z(X,Y)}, \theta\right) = U(X,Y). \tag{2}$$

The function u is a movie : $u(x,y,\theta)$ measures the luminosity (grey-level) of the image point (x,y) at time θ. Taking the derivative of (2) with respect to θ, we obtain the well-known Motion Constraint Equation,

$$u_x \frac{dx}{d\theta} + u_y \frac{dy}{d\theta} + u_\theta = 0. \tag{3}$$

Since $\frac{dy}{d\theta} = 0$ under our hypotheses, this equation defines a scalar apparent velocity on the movie

$$v = \frac{dx}{d\theta} = -\frac{u_\theta}{u_x}, \tag{4}$$

which is related to the scene geometry by

$$v = -\frac{C'(\theta)}{Z}. \tag{5}$$

We can define as well the derivative of any function f along the movement v, also called *total derivative* of f, by

$$\frac{\mathrm{D}f}{\mathrm{D}\theta} = f_\theta + v f_x.$$

As the luminosity of a point of the scene does not change while the camera moves, we have $\frac{\mathrm{D}u}{\mathrm{D}\theta} = 0$, which is nothing but the previous Motion Constraint Equation. We can also compute the apparent acceleration on the movie by

$$? = \frac{\mathrm{D}v}{\mathrm{D}\theta} = v_\theta + v v_x. \tag{6}$$

According to the equations (4) and (5), reconstructing the scene geometry seems easy : first calculate the apparent velocity field of the movie by $v = -\frac{u_\theta}{u_x}$ and then deduce the depth by $Z = -vC'(\theta)$. However, many

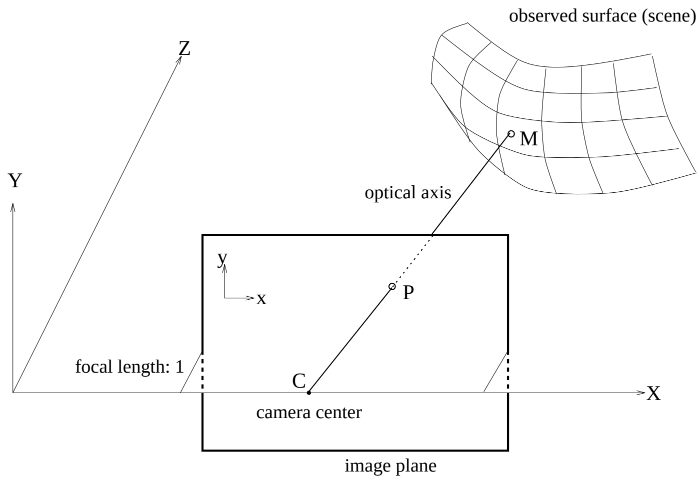

Figure 1: Scene geometry

problems occur if we try to proceed this way : the estimation of v is difficult and very sensitive to noise, and the camera movement, $C(\theta)$, is not always known, so that the redundant depth information expressed along the movie is not easy to collect. In order to solve these difficulties, we need to regularize the movie in a way coherent with the depth recovery. We study in the next section such analyses, using an axiomatic approach.

3 AXIOMATIC FORMULATION

A multiscale analysis of movies is a family of operators $(T_t)_{t\geqslant 0}$ which, applied to an initial movie u_0, leads to filtered versions $u(t) = T_t u_0$ at all scales t. It has been shown[2] that under fundamental hypotheses of locality, causality, and space/time/grey-level translation invariance, multiscale analyses of movies can be described by partial differential equations of the type $\frac{\partial u}{\partial t} = F(D^2u, Du, t)$, with initial condition $u(.,0) = u_0$. In addition to these hypotheses, we constrain the analysis to satisfy several invariance properties.

- **[Morphological Invariance]**. The analysis commutes with any one-to-one grey-level rescaling :

$$\forall h,\ T_t h(u) = h(T_t u).$$

- **[Transversal Invariance]**. The analysis commutes with any nondecreasing y-rescaling :

$$\forall f \nearrow,\ T_t(u \circ R_f) = (T_t u) \circ R_f, \quad \text{where} \quad R_f(x,y,\theta) = (x, f(y), \theta).$$

- **[Galilean Invariance]**. The analysis commutes with the superimposition of any uniform straight translation movement on the movie :

$$\forall \alpha,\ (T_t u) \circ B_\alpha = T_t(u \circ B_\alpha), \quad \text{where} \quad B_\alpha(x,y,\theta) = (x - \alpha\theta, y, \theta).$$

- **[Zoom Invariance]**. The analysis commutes with any spatial zoom :

$$\forall \lambda,\ (T_t u) \circ H_\lambda = T_t(u \circ H_\lambda), \quad \text{where} \quad H_\lambda(x,y,\theta) = (\lambda x, \lambda y, \theta).$$

The morphological invariance and the transversal invariance are natural axioms from the depth recovery point of view, since the associated operators $u \mapsto h(u)$ and R_f leave invariant the estimation of the velocity field, i.e. $v[h(u)] = v[u]$ and $v[u \circ R_f] = v[u] \circ R_f$, where $v[\ ]$ denotes the velocity field operator $u \mapsto -\frac{u_\theta}{u_x}$. The galilean invariance, already used in movie analysis[2], is here adjusted to the camera movement, along the x axis. Last, the zoom invariance states that the analysis will not depend on the focal length of the camera.

THEOREM 3.1. *There is a unique multiscale analysis which satisfies both the fundamental axioms and [Morphological Inv.], [Transversal Inv.], [Galilean Inv.] and [Zoom Inv.]. It is given, up to a rescaling, by the partial differential equation*

$$\frac{\partial u}{\partial t} = u_{\xi\xi} \tag{7}$$

$$\text{with } \xi = (-\frac{u_\theta}{u_x}, 0, 1) \text{ and } u_{\xi\xi} = [D^2 u](\xi, \xi).$$

This theorem propounds a unique depth-compatible multiscale analysis (DCMA) to solve our problem. Note that the evolution equation (7) makes sense in terms of viscosity solutions. The proof of theorem 3.1 has been explained in a previous article[11].

4 PROPERTIES OF THE DEPTH-COMPATIBLE MULTISCALE ANALYSIS (DCMA)

4.1 Diffusion of the apparent movement

THEOREM 4.1. *The DCMA diffuses each component of the movement (velocity $v = \frac{dP}{d\theta}$, acceleration $\frac{d^2P}{d\theta^2}, \ldots$), in the same direction as u, i.e*

$$\frac{\partial}{\partial t}(\frac{d^n P}{d\theta^n}) = (\frac{d^n P}{d\theta^n})_{\xi\xi}.$$

In particular, the apparent velocity v follows an intrinsic, polynomial and causal diffusion equation,

$$\frac{\partial v}{\partial t} = v_{\xi\xi} = v_{\theta\theta} + 2v v_{\theta x} + v^2 v_{xx}.$$

This proposition highlights an interesting property of the DCMA : applying equation (7) on a movie allows to filter indirectly the whole movement field of this movie. The velocity field –what we want to recover finally– is processed in an intrisic way, and therefore two movies of the same scene (and having consequently the same velocity field) would be analyzed the same way.

To prove these properties, it is interesting to introduce the Lie brackets between the partial derivatives $\frac{\partial}{\partial x}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial t}$, which commute together, and the total derivative $\frac{\mathrm{D}}{\mathrm{D}\theta}$. Thus, we define

$$[\frac{\partial}{\partial x}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = \frac{\partial}{\partial x}\frac{\mathrm{D}}{\mathrm{D}\theta} - \frac{\mathrm{D}}{\mathrm{D}\theta}\frac{\partial}{\partial x},$$

which can be effectively computed into

$$[\frac{\partial}{\partial x}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = \frac{\partial}{\partial x}(\frac{\partial}{\partial \theta} + v\frac{\partial}{\partial x}) - (\frac{\partial}{\partial \theta} + v\frac{\partial}{\partial x})\frac{\partial}{\partial x} = v_x \frac{\partial}{\partial x}.$$

As well,

$$[\frac{\partial}{\partial\theta}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = v_\theta \frac{\partial}{\partial x} \quad \text{and} \quad [\frac{\partial}{\partial t}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = v_t \frac{\partial}{\partial x}.$$

Now the notation $f_{\xi\xi} = [D^2 f](\xi,\xi)$ can be clarified :

$$\begin{aligned}
(\ \ )_{\xi\xi} &= \frac{\partial^2}{\partial\theta^2} + 2v\frac{\partial^2}{\partial\theta\partial x} + v^2\frac{\partial^2}{\partial x^2} \\
&= (\frac{\partial}{\partial\theta} + v\frac{\partial}{\partial x})\frac{\partial}{\partial\theta} + v(\frac{\partial}{\partial\theta} + v\frac{\partial}{\partial x})\frac{\partial}{\partial x} \\
&= \frac{\mathrm{D}}{\mathrm{D}\theta}\frac{\partial}{\partial\theta} + v\frac{\mathrm{D}}{\mathrm{D}\theta}\frac{\partial}{\partial x} \\
&= \frac{\mathrm{D}}{\mathrm{D}\theta}(\frac{\partial}{\partial\theta} + v\frac{\partial}{\partial x}) - \frac{\mathrm{D}v}{\mathrm{D}\theta}\frac{\partial}{\partial x}
\end{aligned}$$

and finally

$$(\ \ )_{\xi\xi} = \frac{\mathrm{D}^2}{\mathrm{D}\theta^2} - ?\,\frac{\partial}{\partial x}. \tag{8}$$

In particular, if we call $\psi = \frac{\mathrm{D}\Gamma}{\mathrm{D}\theta}$ the total derivative of $?$, we obtain

$$v_{\xi\xi} = \psi - ?\,v_x. \tag{9}$$

Lemma 4.2. *For any evolution equation,*

$$[\frac{\partial}{\partial t} - (\ \ )_{\xi\xi}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = (v_t - v_{\xi\xi})\frac{\partial}{\partial x}. \tag{10}$$

Proof of the lemma :

We just need to compute the Lie bracket

$$\begin{aligned}
[(\,)_{\xi\xi}, \frac{\mathrm{D}}{\mathrm{D}\theta}] &= [\frac{\mathrm{D}^2}{\mathrm{D}\theta^2} - ?\,\frac{\partial}{\partial x}, \frac{\mathrm{D}}{\mathrm{D}\theta}] \\
&= [\frac{\mathrm{D}^2}{\mathrm{D}\theta^2}, \frac{\mathrm{D}}{\mathrm{D}\theta}] - [?\,\frac{\partial}{\partial x}, \frac{\mathrm{D}}{\mathrm{D}\theta}] \\
&= 0 - ?\,[\frac{\partial}{\partial x}, \frac{\mathrm{D}}{\mathrm{D}\theta}] + \frac{\mathrm{D}?}{\mathrm{D}\theta}\frac{\partial}{\partial x} \\
&= (\psi - ?\,v_x)\frac{\partial}{\partial x} \\
&= v_{\xi\xi}\frac{\partial}{\partial x}
\end{aligned}$$

and by linearity,

$$[\frac{\partial}{\partial t} - (\,)_{\xi\xi}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = [\frac{\partial}{\partial t}, \frac{\mathrm{D}}{\mathrm{D}\theta}] - [(\,)_{\xi\xi}, \frac{\mathrm{D}}{\mathrm{D}\theta}] = (v_t - v_{\xi\xi})\frac{\partial}{\partial x}$$

Proof of the theorem 4.1 :

Applying this lemma to u in the case of the evolution $u_t = u_{\xi\xi}$, we can notice that the left term is null, which proves that $\frac{\partial v}{\partial t} = v_{\xi\xi}$. Consequently, the Lie bracket in (10) is a null operator, so that the diffusion equation extends to all derivatives of v.

4.2 Evolution of the scene interpretation

We now give another property of the DCMA, in terms of scene interpretation. Let us say a movie $u(x, y, \theta)$ is *ideal* if we can find some functions $C(\theta)$, $Z(X, Y)$ and $U(X, Y)$ so that equation (2) holds, meaning that the movie u can be interpreted as a scene $Z(X, Y), U(X, Y)$ (depth and isotropic luminosity) observed by a unit focal length camera under the translation movement $X = C(\theta)$.

THEOREM 4.3. *The DCMA produces, from an ideal movie u_0, a sequence of ideal movies $u(t)$, with the same depth interpretation as u_0, but for which the underlying camera movement is a linearly filtered version of u_0's one. More precisely, the scene space interpretation is given by*

$$Z(X, Y, t) = Z(X, Y, 0) \quad ; \quad \frac{\partial C}{\partial t}(\theta, t) = \frac{\partial^2 C}{\partial \theta^2}.$$

The first equation simply states the conservation of the depth in the case of an ideal movie, which is a kind of perspective invariance : the DCMA preserves the perspective interpretation of a movie. This property is fundamental for our equation : it proves that the concept of ideal movie is compatible with the DCMA. The second one shows that the camera movement is regularized by the heat equation along the analysis. Then we can suppose that this movement is constant after filtering, which makes the reconstruction step more simple.

Remark : It has been proved[11] that the DCMA is the unique equation which satisfies these properties.

Proof :

Given an ideal movie u_0 satisfying

$$u_0 \left(\frac{X - C_0(\theta)}{Z(X, Y)}, \frac{Y}{Z(X, Y)}, \theta \right) = U(X, Y),$$

let us define a sequence of movies $\tilde{u}(t)$ by

$$\tilde{u} \left(\frac{X - C(\theta, t)}{Z(X, Y)}, \frac{Y}{Z(X, Y)}, \theta, t \right) = U(X, Y), \tag{11}$$

where $C(\theta, t)$ is the unique solution of the heat equation $C_t = C''$ with initial condition $C(\theta, 0) = C_0(\theta)$. Taking the derivative of equation (11) with respect to θ and t,

$$-\frac{C'(\theta, t)}{Z(X, Y)} \tilde{u}_x + \tilde{u}_\theta = 0, \tag{12}$$

$$-\frac{C''(\theta, t)}{Z(X, Y)} \tilde{u}_x + \tilde{u}_t = 0. \tag{13}$$

Then, eliminating C'' between (12) and (13), we obtain

$$\tilde{u}_t = \frac{\tilde{u}_x}{Z(X, Y)} \frac{\mathrm{D}}{\mathrm{D}\theta} (Z(X, Y) \frac{\tilde{u}_\theta}{\tilde{u}_x}) = \tilde{u}_x \frac{\mathrm{D}}{\mathrm{D}\theta} (\frac{\tilde{u}_\theta}{\tilde{u}_x}) = \tilde{u}_{\xi\xi}.$$

The movie $\tilde{u}$ is solution of the equation $u_t = u_{\xi\xi}$ with initial condition $u(., 0) = u_0$: since we now that these conditions determine a unique movie, we deduce that $u(t) = \tilde{u}(t)$. We can now conclude that $u(t)$ is ideal for all t (because $\tilde{u}(t)$ is ideal by definition), with depth interpretation $Z(X, Y)$ and filtered camera movement interpretation $C(t)$.

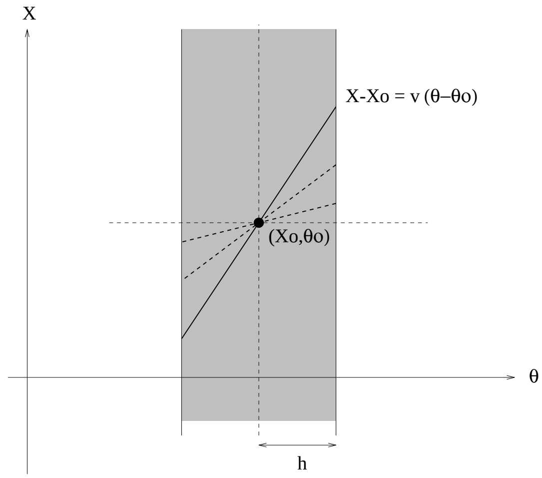

Figure 2: Illustration of the numerical scheme for the DCMA.

4.3 Idealization

THEOREM 4.4. *Let $u(t)$ be a sequence of four times differentiable periodic movies satisfying the DCMA, then the following properties hold :*

1. *$\iint u^2(x,\theta,t)dxd\theta$ does not depend on t.*
2. *$E(t) = \frac{1}{2}\iint ?^2(x,\theta,t)dxd\theta$ decreases with respect to t and $E'(t) = -\iint (\frac{\mathrm{D}\Gamma}{\mathrm{D}\theta})^2 dxd\theta$.*

The first property can be simply interpreted as the conservation of the light energy $\frac{1}{2}\iint u^2 dxd\theta$ by the DCMA. The second one, which states the global decreasing of $?^2$, proves that the DCMA operates an "idealization" on the movie, meaning that as scale t increases, the movie goes closer to an ideal movie with null acceleration. The proof of this theorem is given in appendix A.

5 NUMERICAL SHEME AND EXPERIMENTS

As we said in section 2, the direct estimation of v by equation (5) is quite difficult and too sensitive to noise. To avoid this difficulty, we will use an implicit numerical scheme which does not need to evaluate v properly. Let us consider the following operators (cf. figure. 2) :

$$\begin{aligned}
I_h u(x_0, y_0, \theta_0) &= \inf_{v\in\mathbb{R}} \sup_{-h\leqslant\theta\leqslant h} u(x_0 + v\theta, y_0, \theta_0 + \theta), \\
S_h u(x_0, y_0, \theta_0) &= \sup_{v\in\mathbb{R}} \inf_{-h\leqslant\theta\leqslant h} u(x_0 + v\theta, y_0, \theta_0 + \theta), \\
T_h &= I_h \circ S_h.
\end{aligned}$$

This concept of inf-sup operators has been introduced in mathematical morphology[13]. They are easy to compute, even on a dicrete lattice, and do not require the estimation of image derivatives. One can check easily that the T_h operator given above satisfies the axioms which define the DCMA.

THEOREM 5.1. *Let be $u : \mathbb{R} \mapsto \mathbb{R}$ a 3 times differentiable movie, then in each point where $u_x \neq 0$,*

$$T_h u = u + h^2 u_{\xi\xi} + O(h^3)$$

Thus, iterating T_h allows us to process a real digitized movie by the DCMA in a robust way.

We implemented this scheme in order to produce some numerical experiments (figure 3). We first synthetized two ideal movies, explorations of a half-sphere by a uniform translation camera movement (lines 1 and 5). For the first movie, we took only some fixed points on the half-sphere, whereas the second movie is a continuous one. Then we obtained two noisy movies (lines 2 and 6) by replacing some grey values of the ideal ones by random uniform white noise. Last, we filtered these noisy movies by the discrete scheme of the DCMA at three different scale (lines 3 to 5 for the first movie, lines 8 to 10 for the second one). As expected, we see that the redundancy of the depth information into the noisy movies allows us to remove noise and to recover gradually the ideal movies as the scale increases.

6 CONCLUSION

From an axiomatic point of view, we have studied how to filter movies in a depth-compatible way, in the case of a simple camera motion. We proved that this movie processing focus the coherent and redundant depth information of a movie into a ideal movie of depth-equivalent frames. Thus, we regularize the shape from motion problem by introducing a preprocessing step after which one can apply standard techniques of depth recovery in a more efficient way. This work still needs to be extended to more complex camera movements, in order to be integrated in a real depth-recovery device.

APPENDIX A : PROOF OF THEOREM 4.4

1. *$\iint u^2(x,\theta,t)dxd\theta$ does not depend on t.*

   Let $A(t) = \iint u^2 dxd\theta$, we have

$$\begin{aligned} A'(t) &= 2\iint u u_{\xi\xi} dxd\theta \\ &= -2\iint u ?\, u_x dxd\theta \\ &= -2\iint u u_x (v_\theta + v v_x) dxd\theta \\ &= -2\iint u u_x v_\theta - u u_\theta v_x dxd\theta \end{aligned}$$

   After integration by parts, which eliminates integrated terms because of periodicity,

$$A'(t) = 2\iint (u u_x)_\theta v - (u u_\theta)_x v dxd\theta = 0,$$

   so that $A(t) = A(0)$, which does not depend on t.

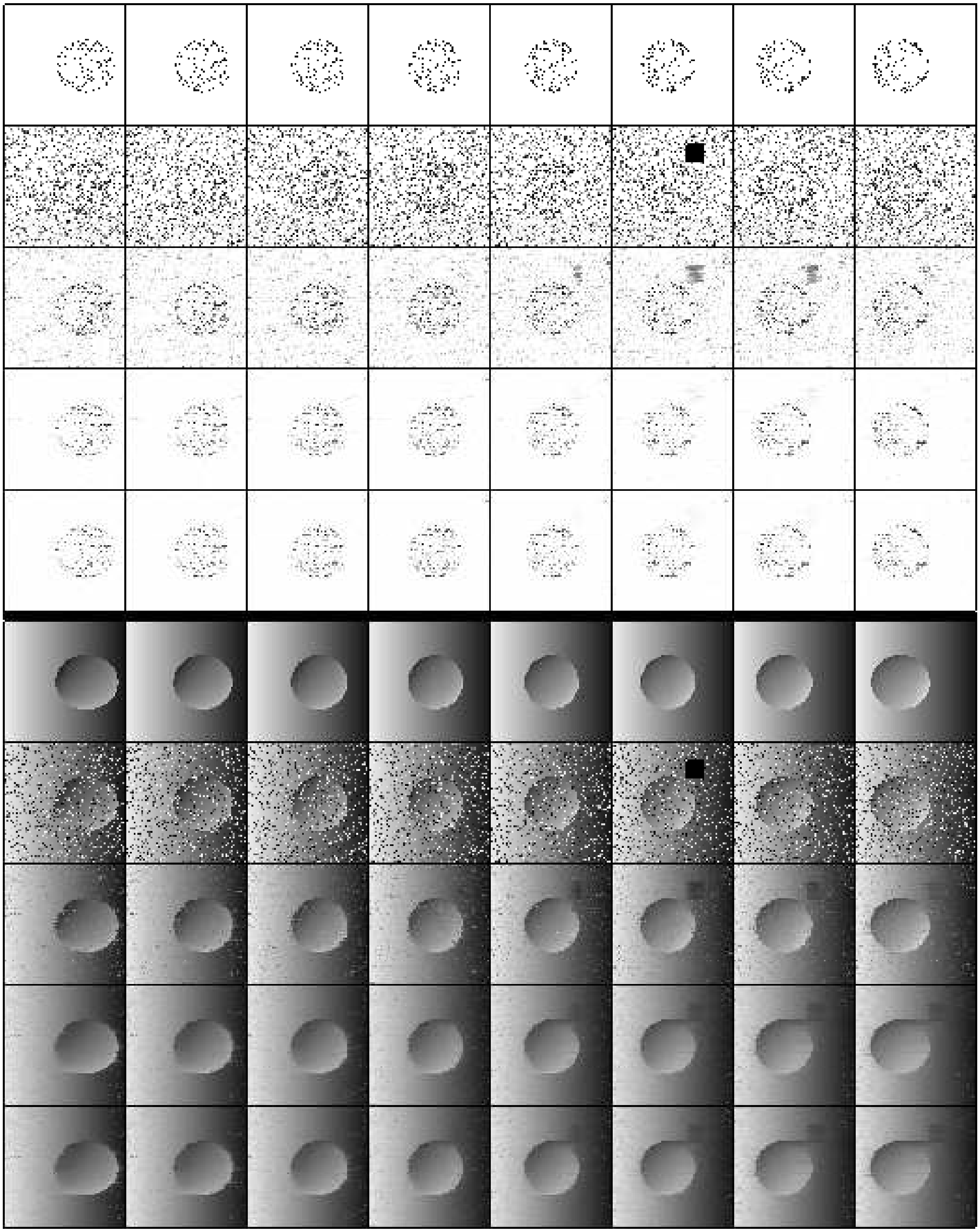

Figure 3: Filtering of a two noisy movies.

2. $E(t) = \frac{1}{2}\iint ?^2(x,\theta,t)dxd\theta$ *decreases with respect to* t *and* $E'(t) = -\iint(\frac{\mathrm{D}\Gamma}{\mathrm{D}\theta})^2 dxd\theta$.

First notice that $?_{\xi\xi} = \frac{\mathrm{D}^2\Gamma}{\mathrm{D}\theta^2} - ??_x = \frac{\mathrm{D}\Psi}{\mathrm{D}\theta} - ??_x$ with $\Psi = \frac{\mathrm{D}\Gamma}{\mathrm{D}\theta} = ?_\theta + v?_x$. Then,

$$\begin{aligned} E'(t) &= \iint ??_{\xi\xi}dxd\theta \\ &= \iint ?(\Psi_\theta + v\Psi_x - ??_x)dxd\theta \\ &= \iint ?\Psi_\theta + (v?)\Psi_x - ?^2?_x.dxd\theta \end{aligned}$$

The integration of the first two terms gives

$$\begin{aligned} E'(t) &= -\iint ?_\theta\Psi + (v?)_x\Psi + ?^2?_x dxd\theta \\ &= -\iint \Psi(?_\theta + v?_x + v_x?) + ?^2?_x dxd\theta \\ &= -\iint \Psi^2 dxd\theta - \iint v_x?\Psi + ?^2?_x dxd\theta. \end{aligned}$$

As $\iint ?^2?_x dxd\theta = \frac{1}{3}\iint \frac{\partial}{\partial x}(?^3)dxd\theta = 0$, the second term can be simplified into

$$\begin{aligned} B(t) = \iint v_x?\Psi + ?^2?_x dxd\theta &= \iint v_x?\Psi - ?^2?_x dxd\theta \\ &= \iint ?(?_\theta v_x + vv_x?_x - ?_x v_\theta - vv_x?_x)dxd\theta \\ &= \frac{1}{2}\iint (2??_\theta)v_x - (2??_x)v_\theta dxd\theta. \end{aligned}$$

After another integration,

$$B(t) = -\frac{1}{2}\iint ?^2(v_{x\theta} - v_{\theta x})dxd\theta = 0,$$

and finally

$$E'(t) = -\iint \Psi^2 dxd\theta < 0.$$

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