Constrained Incomplete Argumentation Frameworks

Jean-Guy Mailly[1[0000−0001−9102−7329]]

LIPADE, University of Paris, France
jean-guy.mailly@u-paris.fr

Abstract. Operations like belief change or merging have been adapted to the context of abstract argumentation. However, these operations may require to express some uncertainty or some disjunction in the result, which is not representable in classical AFs. For this reason, some of these works require a set of AFs or a set of extensions as the outcome of the operation, somehow to represent a disjunction of AFs or extensions. In parallel, the notion of Incomplete AFs (IAFs) has been developed recently. It corresponds to AFs where the existence of some arguments or attacks may be uncertain. Each IAF can be associated with a set of classical AFs called completions, that correspond to different ways of "resolving the uncertainty". While these IAFs could be good candidates for a compact representation of a "disjunction" of AFs, we prove that this model is not expressive enough. Then we introduce Constrained IAFs, that include a propositional formula allowing to select the set of completions used for reasoning. We prove that this model is expressive enough for representing any set of AFs, or any set of extensions. Moreover, we show that the complexity of credulous and skeptical reasoning is the same as in the case of IAFs. Finally, we show that CIAFs can be used to model a new form of extension enforcement.

Keywords: Abstract Argumentation · Uncertainty · Extension Enforcement.

1 Introduction

Representing uncertainty and reasoning with uncertain information is of utmost importance in artificial intelligence. Indeed, there are many reasons that may lead an intelligent agent to face uncertainty or impossibility to choose between alternatives. For instance, she can receive information from different sources, which can have different degrees of reliability. This information can be incompatible with her previous knowledge, or with information provided by other sources. This kind of problem can be formalized as belief change operations ("How to incorporate a new piece of information to my knowledge if it is not logically consistent?") [1, 21, 22] or belief merging ("How to give a coherent representation of several agent's knowledge even if they are globally inconsistent?") [23]. In this kind of application, a simple way to deal with the uncertainty of the result is the logical disjunction: if the result of revising an agent's knowledge is "I am not sure whether a is true or b is true.", then it can be expressed with $a \vee b$. However, there are formalisms where this kind of simple representation of undecidedness cannot be done. For instance, in abstract argumentation frameworks (AFs) [18], either there is certainly an attack between two arguments, or there is certainly no attack between them. But an agent cannot express something like "I am not sure whether a attacks b or not." AFs have been extended in this direction: Partial AFs (PAFs) [9] allow to represent uncertain attacks. Later, Incomplete AFs (IAFs) [6, 5] have been proposed, as a generalization of PAFs where also arguments can be uncertain. Reasoning with a PAF or an IAF is possible thanks to a set of *completions*, that are classical AFs that correspond to the different possible worlds encoded in the uncertain information. While this framework allows to express uncertainty in abstract argumentation in a rich way, there are still situations that cannot be modeled. Consider, *e.g.*, that an agent faces the information "Either a attacks b, or b attacks a, but I am not sure which one is true.". There is no way to represent this information with an IAF. However, this may be necessary in some situations. For instance, several adaptations of belief change [11, 13] or merging [9, 15] to abstract argumentation lead to results that can contain such an uncertainty over the result, impossible to be represented by a single AF. So, these works propose to represent the "disjunction" in the result as a set of AFs, or even as a set of extensions (and it is also known that not every set of extensions can be represented by a single AF [19]).

In this paper, we define a generalization of IAFs, that adds a constraint to it. The constraint in a Constrained IAF (CIAF) is a propositional formula that allows to specify which subset of the completions of the IAF should be used for reasoning. We show that this framework is more expressive than IAFs, in the sense that any set of AFs can be the set of completions of a CIAF. Also, any set of extensions can be obtained from (the completions of) a CIAF. We prove that, despite being more expressive than IAFs, the complexity of credulous and skeptical reasoning does not increase compared to IAFs, under various classical semantics. Interestingly, we also identify a relation between our CIAFs and extension enforcement [3]. This operation consists in modifying an AF s.t. a given set of arguments becomes part of an extension. Classical enforcement operators are based on expansions, *i.e.* addition of arguments and attacks s.t. the attack relation between former arguments remain unchanged. Theoretical results show under which conditions enforcement is possible under expansions. However, these results may suppose the possibility to perform unnatural expansions, like adding a new argument that attacks all the undesired arguments. In a real dialogue, such an "ultimate attacker", that defeats every unwanted argument, is not likely to exist. We show that completions of a CIAF can be used to model the set of expansions that are available to an agent, and then enforcement is possible iff the desired set of arguments is credulously accepted w.r.t. the CIAF.

The paper is organized as follows. Section 2 describes background notions of abstract argumentation. Our first contributions are presented in Section 3 : the definition of CIAFs, the properties of the framework regarding its expressivity, and finally the computational complexity of credulous and skeptical acceptance. Then in Section 4, we show how CIAFs can be used to model scenarios of extension enforcement. We discuss related work in Section 5, and finally Section 6 concludes the paper and highlights some topics of interest for future research.[1]

2 Background

2.1 Dung's Abstract Argumentation

Abstract argumentation was introduced in [18], where arguments are abstract entities whose origin or internal structure are ignored. The acceptance of arguments is purely defined from the relations between them.

Definition 1 (Abstract AF). *An* abstract argumentation framework *(AF) is a directed graph $\mathcal{F} = \langle A, R \rangle$, where A is a set of* arguments*, and $R \subseteq A \times A$ is an* attack relation*.*

We say that a *attacks* b when $(a, b) \in R$. If $(b, c) \in R$ also holds, then a *defends* c against b. Attack and defense can be adapted to sets of arguments: $S \subseteq A$ attacks (respectively defends) an argument $b \in A$ if $\exists a \in S$ that attacks (respectively defends) b.

Example 1. Let $\mathcal{F} = \langle A, R \rangle$ be the AF depicted in Figure 1, with $A = \{a, b, c, d, e\}$ and $R = \{(b, a), (c, a), (c, d), (d, b), (d, c), (e, a)\}$. Each arrow represents an attack. d defends a against both b and c, since these are attackers of a that are, in turn, both attacked by d.

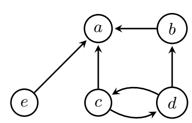

Fig. 1: The AF $\mathcal{F}$

Different semantics have been introduced to evaluate the acceptability of arguments [18], relying on two basic concepts: *conflict-freeness* and *admissibility.*

Definition 2 (Conflict-freeness and Admissibility). *Given $\mathcal{F} = \langle A, R \rangle$, a set of arguments $S \subseteq A$ is:*

- *conflict-free iff $\forall a, b \in S$, $(a, b) \notin R$;*
- *admissible iff it is conflict-free, and defends each $a \in S$ against its attackers.*

[1] Proofs are omitted for space reasons.

We use $\mathrm{cf}(\mathcal{F})$ and $\mathrm{ad}(\mathcal{F})$ for denoting the sets of conflict-free and admissible sets of an argumentation framework $\mathcal{F}$. The intuition behind these principles is that a set of arguments may be accepted only if it is internally consistent (conflict-freeness) and able to defend itself against potential threats (admissibility). The semantics proposed by [18] can be defined as follows.

Definition 3 (Extension Semantics). *Given $\mathcal{F} = \langle A, R \rangle$, an admissible set $S \subseteq A$ is:*

- *a complete extension iff it contains every argument that it defends;*
- *a preferred extension iff it is a $\subseteq$-maximal complete extension;*
- *the unique grounded extension iff it is the $\subseteq$-minimal complete extension;*
- *a stable extension iff it attacks every argument in $A \setminus S$.*

The sets of extensions of an AF $\mathcal{F}$, for these semantics, are denoted (respectively) $\mathrm{co}(\mathcal{F})$, $\mathrm{pr}(\mathcal{F})$, $\mathrm{gr}(\mathcal{F})$ and $\mathrm{st}(\mathcal{F})$. Based on these semantics, we can define the status of any (set of) argument(s), namely *skeptically accepted* (belonging to each σ-extension), *credulously accepted* (belonging to some σ-extension) and *rejected* (belonging to no σ-extension). Given an AF $\mathcal{F}$ and a semantics σ, we use (respectively) $\mathrm{sk}_\sigma(\mathcal{F})$, $\mathrm{cr}_\sigma(\mathcal{F})$ and $\mathrm{rej}_\sigma(\mathcal{F})$ to denote these sets of arguments.

Example 2. We consider again $\mathcal{F}$ given in Figure 1. Its extensions for the different semantics, as well as the sets of accepted arguments, are given in Table 1.

σ	$\sigma(\mathcal{F})$	$\mathrm{cr}_\sigma(\mathcal{F})$	$\mathrm{sk}_\sigma(\mathcal{F})$	$\mathrm{rej}_\sigma(\mathcal{F})$
co	$\{e\}, \{d,e\}, \{b,c,e\}$	$\{b,c,d,e\}$	$\{e\}$	$\{a\}$
pr	$\{d,e\}, \{b,c,e\}$	$\{b,c,d,e\}$	$\{e\}$	$\{a\}$
gr	$\{e\}$	$\{e\}$	$\{e\}$	{a,b,c,d}
st	$\{d,e\}, \{b,c,e\}$	$\{b,c,d,e\}$	$\{e\}$	$\{a\}$

Table 1: Extensions and Accepted Arguments of $\mathcal{F}$ for $\sigma \in \{\mathrm{co}, \mathrm{pr}, \mathrm{gr}, \mathrm{st}\}$

For more details about argumentation semantics, we refer the interested reader to [18, 2].

2.2 Incomplete AFs

Now, we describe Incomplete Argumentation Frameworks [9, 6, 5].

Definition 4 (Incomplete AF). *An* Incomplete Argumentation Framework *(IAF) is a tuple $\mathcal{I} = \langle A, A^?, R, R^? \rangle$, where A and $A^?$ are disjoint sets of arguments, and $R, R^? \subseteq (A \cup A^?) \times (A \cup A^?)$ are disjoint sets of attacks.*

Elements from A and R are certain arguments and attacks, *i.e.* the agent is sure that they appear in the framework. On the opposite, $A^?$ and $R^?$ represent uncertain arguments and attacks. For each of them, there is a doubt about their actual existence.

Example 3. Let us consider $\mathcal{I} = \langle A, A^?, R, R^? \rangle$ given in Figure 2. We use plain nodes and arrows to represent certain arguments and attacks, *i.e.* $A = \{a, b, c, d, e\}$ and $R = \{(b, a), (c, a), (d, b), (d, c)\}$. Uncertain arguments are represented as dashed square nodes (*i.e.* $A^? = \{f\}$) and uncertain attacks are represented as dotted arrows (*i.e.* $R^? = \{(e, a), (f, d)\}$).

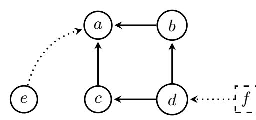

Fig. 2: The IAF $\mathcal{I}$

The notion of completion in abstract argumentation was first defined in [9] for Partial AFs (*i.e.* IAFs with $A^? = \emptyset$), and then adapted to IAFs. Intuitively, a completion is a classical AF which describes a situation of the world coherent with the uncertain information encoded in the IAF.

Definition 5 (Completion of an IAF). *Given* $\mathcal{I} = \langle A, A^?, R, R^? \rangle$*, a completion of* $\mathcal{I}$ *is* $\mathcal{F} = \langle A', R' \rangle$*, s.t.* $A \subseteq A' \subseteq A \cup A^?$ *and* $R_{|A'} \subseteq R' \subseteq R_{|A'} \cup R^?_{|A'}$*, where* $R_{|A'} = R \cap (A' \times A')$ *(and similarly for* $R^?_{|A'}$*).*

The set of completions of an IAF $\mathcal{I}$ is denoted $\mathrm{comp}(\mathcal{I})$.

Example 4. We consider again the IAF from Figure 2. Its set of completions is described at Figure 3.

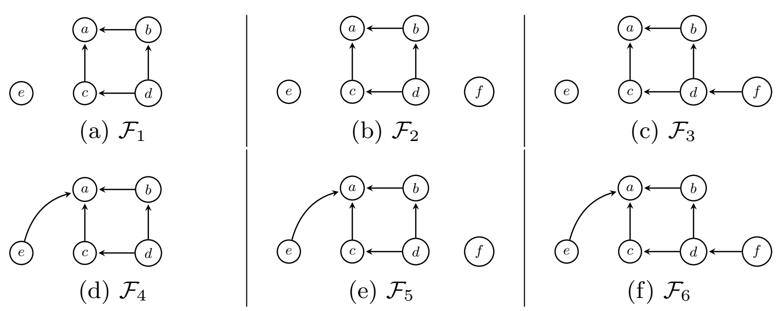

Fig. 3: The Completions of $\mathcal{I}$

Concerning the question of compact representation of a set of AFs by means of an incomplete AF, the following example proves that some sets of AFs (even simple ones) cannot be represented by an IAF.

Example 5. Suppose that the result of revising an AF [11] is the set $\mathfrak{F} = \{\mathcal{F}_1 = \langle\{a,b\},\{(b,a)\}\rangle, \mathcal{F}_2 = \langle\{a,c\},\{(c,a)\}\rangle\}$. The question is to determine whether this set can be compactly represented by a single IAF. Towards a contradiction, suppose that there is an IAF $\mathcal{I} = \langle A, A^?, R, R^?\rangle$ s.t. $\mathrm{comp}(\mathcal{I}) = \mathfrak{F}$. Since a belongs to both $\mathcal{F}_1$ and $\mathcal{F}_2$, it must belong to the certain arguments A. On the contrary, the uncertain arguments are $A^? = \{b,c\}$, each of them belongs to some (but not all) completions. $A = \{a\}$ and $A^? = \{b,c\}$ imply the existence of some completions that only contain a, and some completions that contain the three arguments a,b,c. This is not the case. So $\mathcal{I}$ does not exist.

3 Constrained IAFs

Now we introduce the Constrained Incomplete Argumentation Frameworks, that generalize IAFs by adding a constraint on the set of possible completions.

3.1 Constraints on Completions

Intuitively, for a given $\mathcal{I}$, a constrained version of it is a pair $\langle\mathcal{I}, C\rangle$ where $C \subseteq \mathrm{comp}(\mathcal{I})$. Then, reasoning on $\langle\mathcal{I}, C\rangle$ requires to use only C instead of the full set of completions of $\mathcal{I}$. For instance, let us consider applications where the uncertainty about the world is encoded as a set of AFs or a set of extensions (for instance, revision [11, 13] or merging of argumentation frameworks [9, 15]). This set of AFs or extensions may not be encodable in a single IAF, while being representable with a single Constrained IAF. But rather than defining the constraint with a set of completions, we define a logical language to express information on the structure of an AF, *i.e.* a propositional language s.t. the models of a formula correspond to AFs, inspired by [10] for selecting extensions. This is a more compact representation of the constraint.

Definition 6 (Constraint). *Given A a set of arguments, we define the set of propositional atoms $Prop_A = Arg_A \cup Att_A$ where $Arg_A = \{\mathrm{arg}_a \mid a \in A\}$ and $Att_A = \{\mathrm{att}_{a,b} \mid (a,b) \in A \times A\}$. Then, $\mathcal{L}_A$ is the propositional language built from $Prop_A$ with classical connectives $\{\neg, \vee, \wedge\}$.*

The satisfaction of a constraint by an AF is defined as follows.

Definition 7 (Constraint Satisfaction). *Given A a set of arguments, and $\phi \in \mathcal{L}_A$ a formula, the set of models of ϕ is denoted* $\mathrm{mod}(\phi)$*. An AF $\mathcal{F} = \langle A', R\rangle$ with $A' \subseteq A$ and $R \subseteq A' \times A'$* satisfies *ϕ iff there is a model $\omega \in \mathrm{mod}(\phi)$ s.t. $A' = \{a \in A \mid \omega(\mathrm{arg}_a) = \top\}$, and $R = \{(a,b) \in A \times A \mid \omega(\mathrm{att}_{a,b}) = \top)\}$.*

3.2 Definition and Expressivity of CIAFs

Definition 8 (Constrained IAF). *A* Constrained Incomplete Argumentation Framework *(CIAF) is a tuple $\mathcal{C} = \langle A, A^?, R, R^?, \phi\rangle$, where $\langle A, A^?, R, R^?\rangle$ is an IAF, and $\phi \in \mathcal{L}_{A \cup A^?}$ is a constraint.*

The constraint ϕ is used to select a subset of the completions of the IAF $\mathcal{I}_\mathcal{C} = \langle A, A^?, R, R^? \rangle$. The completions of a CIAF are then defined as follows.

Definition 9 (Completions of a CIAF). *Given* $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$ *a CIAF, we define its set of completions by* $\mathrm{comp}(\mathcal{C}) = \{c \in \mathrm{comp}(\mathcal{I}_\mathcal{C}) \mid c \text{ satisfies } \phi\}$ *where* $\mathcal{I}_\mathcal{C} = \langle A, A^?, R, R^? \rangle$.

Example 6. Let $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$ be a CIAF s.t. $\mathcal{I}_\mathcal{C} = \langle A, A^?, R, R^? \rangle$ is the IAF from Figure 2, and $\phi = \mathrm{att}_{e,a} \wedge \mathrm{arg}_f$. Recall that the completions of $\mathcal{I}_\mathcal{C}$ are given in Figure 3. Only two of them satisfy ϕ, namely $\mathcal{F}_5$ (Fig. 3e) and $\mathcal{F}_6$ (Fig. 3f). So $\mathrm{comp}(\mathcal{C}) = \{\mathcal{F}_5, \mathcal{F}_6\}$.

Let us mention that, in order to be meaningful, the constraint ϕ must satisfy some conditions. Indeed, there must be at least one model of ϕ s.t. arg_a is true for each $a \in A$, $\mathrm{att}_{a,b}$ is true for each $(a,b) \in R$, and $\mathrm{att}_{a,b}$ is false for each $(a,b) \in ((A \cup A') \times (A \cup A')) \setminus (R \cup R^?)$. Otherwise, $\mathrm{comp}(\mathcal{C})$ is trivially empty. More generally, a CIAF $\mathcal{C}$ is *over-constrained* when $\mathrm{comp}(\mathcal{C}) = \emptyset$.

Now, we focus on the expressivity of CIAFs, *i.e.* given a set of AFs (or a set of extensions), is there a CIAF s.t. its completions (or the extensions of its completions) correspond to the given set? We show that, in both cases, the answer is yes.

Representing a Set of AFs First, we define a particular formula, that is only satisfied by one given AF.

Definition 10. *Given* A *a set of arguments, and* $\mathcal{F} = \langle A', R \rangle$ *with* $A' \subseteq A$, *and* $R \subseteq A' \times A'$, *we define* $\psi_\mathcal{F} \in \mathcal{L}_A$ *as*

$$\psi_\mathcal{F} = (\bigwedge_{a \in A'} \mathrm{arg}_a) \wedge (\bigwedge_{a \in A \setminus A'} \neg \mathrm{arg}_a) \wedge (\bigwedge_{(a,b) \in R} \mathrm{att}_{a,b}) \wedge (\bigwedge_{(a,b) \in (A \times A) \setminus R} \neg \mathrm{att}_{a,b})$$

Proposition 1. *Let* $\mathfrak{F} = \{\mathcal{F}_1 = \langle A_1, R_1 \rangle, \dots, \mathcal{F}_n = \langle A_n, R_n \rangle\}$ *be a set of AFs. There is a CIAF* $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$ *s.t.* $\mathrm{comp}(\mathcal{C}) = \mathfrak{F}$.

Intuitively, a simple CIAF that does the job consists of all the arguments and attacks from $\mathfrak{F}$ defined as uncertain, and then ϕ is the disjunction of the $\psi_\mathcal{F}$ formulas, for $\mathcal{F} \in \mathfrak{F}$. Let us exemplify this result.

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Example 7. We continue Example 5. For $\mathfrak{F} = \{\mathcal{F}_1 = \langle \{a, b\}, \{(b, a)\} \rangle, \mathcal{F}_2 = \langle \{a, c\}, \{(c, a)\} \rangle\}$, we define $\mathcal{C} = \langle A, A^?, R, R^?, \phi, \rangle$, with $A = \emptyset$, $A^? = \{a, b, c\}$, $R = \emptyset$, $R^? = \{(b, a), (c, a)\}$, $\phi = \psi_{\mathcal{F}_1} \vee \psi_{\mathcal{F}_2}$, where

$$\psi_{\mathcal{F}_1} = \mathrm{arg}_a \wedge \mathrm{arg}_b \wedge \neg \mathrm{arg}_c \wedge \mathrm{att}_{b,a} \wedge (\bigwedge_{(x,y) \in (\{a,b,c\} \times \{a,b,c\}) \setminus \{(b,a)\}} \neg \mathrm{att}_{x,y})$$

and

$$\psi_{\mathcal{F}_2} = \mathrm{arg}_a \wedge \neg \mathrm{arg}_b \wedge \mathrm{arg}_c \wedge \mathrm{att}_{c,a} \wedge (\bigwedge_{(x,y) \in (\{a,b,c\} \times \{a,b,c\}) \setminus \{(c,a)\}} \neg \mathrm{att}_{x,y})$$

We have $\mathrm{comp}(\mathcal{C}) = \mathfrak{F}$.

Representing a Set of Extensions Now, we focus on the expressibility of a set of extensions with a CIAF.

Proposition 2. *Let* $\mathfrak{E} = \{E_1, \dots, E_n\}$ *be a set of extensions, and* $\sigma \in \{\text{co}, \text{pr}, \text{st}, \text{gr}\}$. *There is a CIAF* $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$ *s.t.* $\bigcup_{c \in \text{comp}(\mathcal{C})} \sigma(c) = \mathfrak{E}$.

Of course, the construction described in Example 7 only shows the existence of a CIAF that satisfies the expected property. This does not mean that this given CIAF is the best way to represent the set of AFs (or extensions). A first possible simplification consists in choosing $A = \bigcap_{i=1}^{n} A_i$ and $A^? = \bigcup_{i=1}^{n} A_i \setminus A$. This natural simplification means that an argument that appears in every AF must be considered as certain. A similar reasoning can be made with the attacks, and the constraint can also be simplified. In a context of belief revision [11, 13] or belief merging [9, 15], it is important to ensure that the resulting CIAF is as close as possible to the initial AF(s). This question is out of the scope of this paper, and is kept for future research.

3.3 Complexity Issues

Observation 1 *Verifying whether an AF (or a completion) satisfies a constraint* ϕ *is a polynomial task: the correspondence between an AF and an interpretation* ω *described in Definition 7 can be done polynomially, as well as checking whether* ω *satisfies* ϕ.

This means that, given a CIAF $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$, guessing a completion of $\mathcal{C}$ is equivalent to guessing a set of arguments $A \subseteq A' \subseteq A^?$, a set of attacks $R_{|A'} \subseteq A' \subseteq R^?_{|A'}$, and verifying (in polynomial time) whether $\langle A', R' \rangle$ satisfies ϕ. This will be useful in the proofs of complexity results.

Now we study the complexity of credulous and skeptical reasoning. More specifically, given $\mathcal{C} = \langle A, A^?, R, R^?, \phi \rangle$ a CIAF, $a \in A$, and σ a semantics,

$\mathsf{Cred}\text{-}\sigma$ is $a \in \bigcup_{\mathcal{F} \in \text{comp}(\mathcal{C})} \bigcup_{S \in \sigma(\mathcal{F})} S$?
$\mathsf{Skep}\text{-}\sigma$ is $a \in \bigcap_{\mathcal{F} \in \text{comp}(\mathcal{C})} \bigcap_{S \in \sigma(\mathcal{F})} S$?

These problems correspond to *possible credulous acceptance* (PCA) and *necessary skeptical acceptance* (NSA) for IAFs [5]. We prove that complexity does not increase from IAFs to CIAFs.

Proposition 3. *The following hold:*

1. *For* $\sigma \in \{\text{ad}, \text{st}, \text{co}, \text{gr}, \text{pr}\}$, $\mathsf{Cred}\text{-}\sigma$ *is* NP*-complete.*
2. *For* $\sigma \in \{\text{st}, \text{co}, \text{gr}\}$, $\mathsf{Skep}\text{-}\sigma$ *is* coNP*-complete.*
3. $\mathsf{Skep}\text{-pr}$ *is* Π_2^P*-complete.*

Concerning skeptical reasoning, the problem is trivial under $\sigma = \text{ad}$, as usual, since $\emptyset$ is admissible in any AF, there is no skeptically accepted argument in any completion.

Finally, let us mention that credulous and skeptical acceptance can be generalized to sets of arguments. These versions consists, respectively, in determining whether a given set of arguments S satisfies $S \subseteq \bigcup_{\mathcal{F} \in \text{comp}(\mathcal{C})} \bigcup_{S \in \sigma(\mathcal{F})} S$, or $S \subseteq \bigcap_{\mathcal{F} \in \text{comp}(\mathcal{C})} \bigcap_{S \in \sigma(\mathcal{F})} S$. The generalized versions keep the same complexity.

4 CIAFs and Extension Enforcement

4.1 Expansion-based Enforcement

Now we introduce the notions of AF expansion and extension enforcement [3].

Definition 11. *Let $\mathcal{F} = \langle A, R\rangle$ be an AF. An* expansion *of $\mathcal{F}$ is an AF $\mathcal{F}' = \langle A \cup A', R \cup R'\rangle$ s.t. $A' \neq \emptyset$ and $A \cap A' = \emptyset$. An expansion is called* normal *if $\forall(a, b) \in R'$, $a \in A'$ or $b \in A'$. Moreover, a normal expansion is* strong *(resp.* weak*) if $\forall(a, b) \in R'$, $a \notin A$ (resp. $b \notin A$).*

In words, an expansion adds some arguments, and possibly attacks. In the case of a normal expansion, the only added attacks concern at least one new arguments, *i.e.* the attacks between the former arguments are not modified. Finally, a normal expansion is strong (resp. weak) if it adds only strong (resp. weak) arguments, *i.e.* arguments that are not attacked by (resp. do not attack) the former arguments. The fact that $\mathcal{F}'$ is an expansion of $\mathcal{F}$ is denoted $\mathcal{F} \preceq_E \mathcal{F}'$ (and normal, strong, weak expansions are denoted by $\preceq_N, \preceq_S, \preceq_W$).

Definition 12. *Given $\mathcal{F} = \langle A, R\rangle$, a set of arguments $S \subseteq A$, and a semantics σ, the AF $\mathcal{F}'$ is a normal (resp. strong, weak) σ-enforcement of S in $\mathcal{F}$ iff $\mathcal{F}'$ is a normal (resp. strong, weak) expansion of $\mathcal{F}$, and $\exists E \in \sigma(\mathcal{F}')$ s.t. $S \subseteq E$.*

Definition 12 only considers "non-strict" enforcement, *i.e.* the desired set of arguments must be included in an extension of the new AF. Strict enforcement is defined in a similar manner, but the desired set of arguments must exactly correspond to an extension.

Some (im)possibility results for these operations have been presented in [3]. However, some results rely on examples that are not representative of realistic argument-based dialogues. The following example is inspired by [3, Theorem 4].

Example 8. Let $\mathcal{F} = \langle A, R\rangle$ be the AF given in Figure 1. Recall that its stable extensions are $\mathrm{st}(\mathcal{F}) = \{\{d, e\}, \{b, c, e\}\}$. Now let $S = \{a, d\}$ be the set of arguments to be enforced. We can define the (strong) expansion $\mathcal{F}' = \langle A \cup \{x\}, R \cup R'\rangle$ where x is a fresh argument, and $R' = \{(x, y) \mid y \in A \setminus S\}$. $\mathcal{F}'$ is shown at Figure 4. $\mathrm{st}(\mathcal{F}') = \{\{x, a, d\}\}$, thus it is a strong enforcement of S in $\mathcal{F}$.

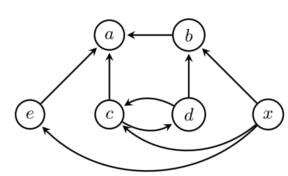


Fig. 4: The Expansion $\mathcal{F}'$ Enforces $S = \{a, d\}$

Example 8 illustrates the (theoretical) possibility to enforce any (conflict-free) set of arguments if strong (or normal) expansions are permitted. However, in an application context like dialogue (*e.g.* argument-based negotiation [17] or persuasion [7]), the existence of an "ultimate" attacker like x, that defeats all the undesired arguments, is unlikely.

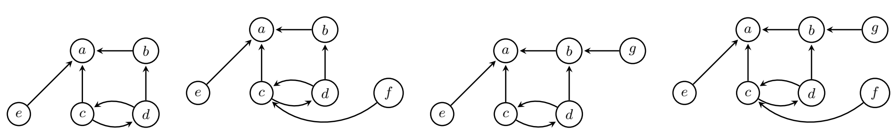
4.2 Enforcement as Credulous Acceptability in CIAFs

To handle the problem highlighted by Example 8, we propose to take into account the set of arguments and attacks that an agent has at her disposal for participating to the debate. This means that we parameterize the expansion operation by the set of possible expanded AFs resulting of using some of the available arguments and attacks.

Definition 13. *Given $\mathcal{F} = \langle A, R \rangle$ an AF, $\mathcal{A}$ a set of available arguments s.t. $A \cap \mathcal{A} = \emptyset$, and $\mathcal{R} \subseteq ((A \cup \mathcal{A}) \times (A \cup \mathcal{A})) \setminus (A \times A)$, we say that $\mathcal{F}' = \langle A', R' \rangle$ is an $\mathcal{A}$-$\mathcal{R}$-parameterized expansion of $\mathcal{F}$ (denoted by $\mathcal{F} \preceq^{\mathcal{A},\mathcal{R}} \mathcal{F}'$) iff $\mathcal{F} \preceq_E \mathcal{F}'$, $A \subseteq A' \subseteq A \cup \mathcal{A}$ and $R' = (R \cup \mathcal{R}) \cap (A' \times A')$.*

We use $\preceq_N^{\mathcal{A},\mathcal{R}}$ (resp. $\preceq_S^{\mathcal{A},\mathcal{R}}$, $\preceq_W^{\mathcal{A},\mathcal{R}}$) to denote $\mathcal{A}$-$\mathcal{R}$-parameterized normal (resp. strong, weak) expansions, *i.e.* $\mathcal{A}$-$\mathcal{R}$-parameterized expansions where $\mathcal{F}'$ is (additionally) normal (resp. strong, weak). This definition allows to take into account the arguments and attacks that are actually known by an agent that participates in a debate. We can show that a set of arguments that can be enforced with an arbitrary (strong) expansion (like in Example 8) may not be enforceable with parameterized expansions.

Example 9. We continue Example 8. Suppose that the available arguments and attacks are $\mathcal{A} = \{f, g\}$ and $\mathcal{R} = \{(f, c), (g, b)\}$. Figure 5 depicts the agent's possible actions: say nothing (*i.e.* keep the initial AF, Fig. 5a), say "f attacks c" (Fig 5b), say "g attacks b" (Fig 5c), or both (Fig. 5d). In all the possible cases, $S = \{a, d\}$ is not enforced, since a is never defended against e.



(a) (b) (c) (d)

Fig. 5: The Agent's Possible Actions

What we call here the "possible actions" of the agent can actually be seen as the set of completions of a CIAF, and the possibility of enforcing a set of arguments corresponds to the credulous acceptance of this set w.r.t. the CIAF.

Definition 14. *Given $\mathcal{F}$ an AF, $\mathcal{A}$ a set of arguments, $\mathcal{R}$ a set of attacks, and $X \in \{E, N, S, W\}$ denoting the type of expansion, we define $\mathfrak{F} = \{\mathcal{F}\} \cup \{\mathcal{F}' \mid \mathcal{F} \preceq_X^{\mathcal{A},\mathcal{R}} \mathcal{F}'\}$. Then, $\mathcal{C}_{\mathcal{F},X}^{\mathcal{A},\mathcal{R}}$ is a CIAF s.t.* $\text{comp}(\mathcal{C}_{\mathcal{F},X}^{\mathcal{A},\mathcal{R}}) = \mathfrak{F}$.

The existence of $\mathcal{C}_{\mathcal{F},X}^{\mathcal{A},\mathcal{R}}$ is guaranteed by Proposition 1. The construction illustrated by Example 7 provides a suitable $\mathcal{C}_{\mathcal{F},X}^{\mathcal{A},\mathcal{R}}$. However, other CIAFs can be defined, for instance all the arguments and attacks from the initial $\mathcal{F}$ can be defined as certain elements. The following proposition states that CIAFs can be used as a computational tool for determining the possibility of enforcement.

Proposition 4. *Given an AF $\mathcal{F} = \langle A, R\rangle$, a set of arguments $S \subseteq A$, $X \in \{E, N, S, W\}$, $\mathcal{A}$ a set of arguments and $\mathcal{R}$ a set of attacks, and a semantics σ, S can be σ-enforced in $\mathcal{F}$ by means of a $\mathcal{A}$-$\mathcal{R}$-parameterized X-expansion iff S is credulously accepted in $\mathcal{C}_{\mathcal{F},X}^{\mathcal{A},\mathcal{R}}$ w.r.t. σ.*

Observe that this result holds for non-strict enforcement, as given in Definition 12. Strict enforcement requires, instead, the notion of extension verification [20] for CIAFs, *i.e.* we must check that the set of arguments S is actually an extension of one of the completions.

5 Related Work

We have described the main existing work on IAFs. Let us also mention [20], which defines an alternative notion of extension (compared to the one from [6]). This does not have an impact on the work presented here, since we focus on argument acceptance; our definitions are consistent with the ones in [5]. However, as mentioned previously, this will be useful for characterizing strict extension enforcement with CIAF-based reasoning.

Besides IAFs, our contribution is related to other previous works. Using propositional formulas as constraints in an argumentation framework has been originally proposed in [10], which defines Constrained Argumentation Frameworks. In this setting, the propositional formula is a constraint on arguments that is used for selecting the best extensions. Intuitively, we use here the constraint in CIAFs in a similar way, but for selecting completions of a IAF instead of selecting the extensions of a (classical) AF.

We have shown how to represent any set of extensions with a single CIAF. The question of representing sets of extensions has already arisen in classical AFs. This corresponds to the notion of realizability in the literature [4, 19], *i.e.* given a set of extensions E and a semantics σ, is there an AF $\mathcal{F}$ s.t. $\sigma(\mathcal{F}) = E$. Existing results show that it is not possible in general for most classical semantics. The non-realizability of some sets of extensions is the reason why some operations (like belief revision or merging) cannot be easily adapted to AFs, as mentioned in the introduction. With Proposition 2, we continue this line of research, by proving the realizability of any set of extensions by means of CIAFs.

Regarding extension enforcement, it has been proven that (non-strict) enforcement is NP-complete [28] for another type of authorized change: argument-fixed enforcement [12], where the set of arguments cannot be modified, but all the attacks (or non-attacks) can be questioned. Although this is out of the scope of this paper, we believe that this kind of enforcement can also be captured by the CIAF setting, which will allow to define a parameterized version of argument-fixed enforcement. The parameters $\mathcal{A}$ and $\mathcal{R}$ are also reminiscent of the "control part" of Control AFs [16, 25, 24], that allows to enforce a set of arguments in presence of uncertainty.

Constraints that express dependencies between arguments of an ADF [8] in a dynamic context have been studied in [27]. While there is some similarity between these constraints and the ones defined here, both studies have different purposes. Indeed, [27] does not focus on uncertain environment as we do here, but only on dynamic scenarios. Connections with enforcement based on $\mathcal{A}$-$\mathcal{R}$-parameterized expansions will be studied.

6 Conclusion

We have defined Constrained Incomplete Argumentation Frameworks (or CIAFs, for short) that generalize IAFs by adding a constraint over the set of completions. This new framework increases the expressivity of IAFs without a gap in complexity, and paves the way for the definition of revision or merging operators for AFs that return a CIAF, *i.e.* a more compact result than a (potentially exponentially large) set of AFs or extensions. However, the CIAF that we have exhibited here to prove the representability of any set of AFs or extensions may not be a suitable solution in scenarios like belief revision or belief merging, where the notion of minimal change is important. We will study how to generate a CIAF that is optimal in such contexts. Knowledge compilation [14] is an interesting way for providing a succinct equivalent propositional constraint such that relevant reasoning tasks are polynomially doable. Other interesting research tracks are the study of complexity for other decision problems (*e.g.* extension verification, or possible skeptical and necessary credulous acceptance), and the implementation of efficient algorithms (*e.g.* based on Boolean encoding, in the line of [26]). We will also study how to encode other extension enforcement operators as CIAF-based reasoning.

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