On Incompleteness in Abstract Argumentation: Complexity and Expressiveness

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Abstract. One of the recent trends in research about abstract argumentation is the study of how incomplete knowledge can be integrated to argumentation frameworks (AFs). In this paper, we survey main results on Incomplete AFs (IAFs), following two directions: how hard is it to reason with IAFs? And what can be expressed with IAFs? We show that two generalizations of IAFs, namely Rich IAFs and Constrained IAFs, despite having a higher expressive power than IAFs, have the same complexity regarding classical reasoning tasks.

Keywords: Abstract argumentation · Uncertainty · Incomplete knowledge · Computational complexity

1 Introduction

Abstract argumentation [18] has been a prominent formalism in the domain of Knowledge Representation and Reasoning, allowing an elegant representation of conflicting information. Classically, an argumentation framework (AF) is a directed graph where the nodes are *arguments* and the edges are *attacks* between these arguments. Reasoning is then based on the selection of sets of arguments that can be collectively accepted, named *extensions*. Since the seminal paper by Dung, various generalizations of the original framework have been proposed (using weights on attacks [20] or arguments [34], preferences [1], collective attacks [32], ...) as well as new reasoning methods [8].

We focus on one such generalization of Dung's framework, namely Incomplete Argumentation Frameworks (IAFs) [3,6,26], where both arguments and attacks can have two different natures: either they are *certain* or they are *uncertain*. While there exists models where this uncertainty is quantified (mainly, with probabilities of existence attached to the elements [25]), in IAFs the uncertainty is purely qualitative: uncertain elements are maybe actually there, maybe not, but the agent reasoning with such an IAF does no have more information about the uncertain elements. This kind of uncertainty in abstract argumentation can be intuitively justified in various ways. An argument or attack can be uncertain, for instance, in multi-agent contexts where one agent tries to model the knowledge of other agents. Then, it is a reasonable assumption that an agent does

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not perfectly know the internal state of other agents. This means that an argument can be uncertain in situations where the agent is not sure whether other agents know this argument (or whether they will choose to use it). Similarly, an agent usually does not perfectly know the preferences of other agents. So, if an agents knows that there is a conflict between two arguments a and b, but she does not know whether her opponent prefers a to b, or b to a, then she does not know whether there is actually an attack between these arguments in her opponent's internal knowledge (in the spirit of Preference-based AFs, where attacks are somehow "cancelled" when they are contradicted by preferences [1]). This has motivated the use of IAFs (or more precisely, Control AFs [15,28], a generalization of IAFs) for defining automated negotiation protocols where agents have partial knowledge about their opponent [16,17].

In this paper, we present the main results in the literature about the complexity of reasoning with IAFs. We describe two families of approaches for defining the acceptable arguments with respect to an IAF. The first one is based on the notion of *completion* (*i.e.* standard AFs that represent, roughly speaking, the possible worlds compatible with the uncertain information encoded in the IAF), and the second one is based on adaptation to IAFs of the basic principles underlying classical AF semantics, namely conflict-freeness and defense. We show how hard it is to reason with IAFs, compared to the classical reasoning approach for standard AFs. Then, we focus on the expressivity of IAFs. More precisely, we answer the question "Can any set of AFs correspond to the set of completions of an IAF?". We show that it is not the case. A partial solution to increase the expressivity of the formalism is to add another kind of uncertainty in the model: uncertainty about the direction of attacks. This model is called Rich IAF [27]. However, even this solution does not allow to represent any set of completions. Then, we propose the Constrained IAFs [29], where an IAF is attached with a propositional formula describing the "valid" completions, *i.e.* any completion not complying with the formula is not used for defining reasoning methods. We show that this model allows to represent any set of completions, and in turn this is a powerful tool for solving representation problems about extensions, in the context of belief revision or belief merging applied to abstract argumentation [11,14]. Finally, we describe some challenges about the CIAF model, regarding the construction of an optimal CIAF for representing a given set of completions (or extensions), where optimality can concern either the graph part of the CIAF, or the syntax of the propositional constraint.

2 Background: Abstract Argumentation Frameworks

Roughly speaking, abstract argumentation [18] is the study of how one can conclude a reasonable point of view about conflicting pieces of information. Usually, an *abstract argumentation framework* is simply a directed graph where the nodes represent the pieces of information and the edges represent the conflicts between them. The exact nature of these pieces of information is ignored. We follow this approach in this paper, and we assume the existence of a finite set $\mathbf{A}$ of atomic entities called arguments. An argumentation framework is then a directed graph whose nodes are a subset of $\mathbf{A}$.

Definition 1 (Argumentation Framework [18]). *An argumentation framework (AF) is a pair* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ *with* $\mathcal{A} \subseteq \mathbf{A}$ *the set of* arguments *and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *the set of* attacks*.*

For $a, b \in \mathcal{A}$, we say that a *attacks* b if $(a, b) \in \mathcal{R}$. If b attacks some $c \in \mathcal{A}$, then a *defends* c against b. Similarly, a set $S \subseteq \mathcal{A}$ attacks an argument b if there is some $a \in S$ that attacks b. Finally, S defends some $b \in \mathcal{A}$ if S attacks all the attackers of b.

Example 1. In the AF $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$, with $\mathcal{A} = \{a, b, c, d, e, f, g\}$ and $\mathcal{R} = \{(a, b), (b, c), (c, d), (d, c), (d, e), (e, f), (f, g), (g, e)\}$ (see Fig. 1), c is attacked by b and d, and it is defended by $\{a, c\}$ (because $\{a, c\}$ attacks b and d, more precisely a attacks b and c attacks d).

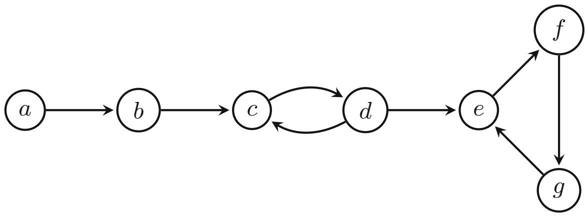

Fig. 1. An example of AF $\mathcal{F}$

In the seminal paper on abstract argumentation [18], Dung proposes a family of methods to reason with an AF, based on the notion of *extension*. An extension is a set of arguments that can be jointly accepted. The methods for determining the extensions are called *semantics*, and they are usually based on two principles: conflict-freeness and admissibility.

Definition 2 (Conflict-freeness and Admissibility). *Given* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ *an AF, the set* $S \subseteq \mathcal{A}$ *is*

- conflict-free *iff* $\forall a, b \in S$, $(a, b) \notin \mathcal{R}$;
- admissible *iff it is conflict-free and* $\forall a \in S$, $\forall b \in \mathcal{A}$ *s.t.* $(b, a) \in \mathcal{R}$, $\exists c \in S$ *s.t.* $(c, b) \in \mathcal{R}$.

Intuitively, a set of arguments is admissible if it is a point of view on the AF which is internally coherent and can defend itself against all the attacks it receives. We use $\mathsf{cf}(\mathcal{F})$ (respectively $\mathsf{ad}(\mathcal{F})$) to denote the set of conflict-free (respectively admissible) sets of an AF $\mathcal{F}$. These basic principles are used to define admissibility-based semantics as follows:

Definition 3 (Admissibility-based Semantics). *Given* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ *an AF, the admissible set* $S \subseteq \mathcal{A}$ *is*

- *a* complete *extension iff S contains all the arguments that it defends;*
- *a* preferred *extension iff S is a $\subseteq$-maximal admissible set;*
- *a* grounded *extension iff S is a $\subseteq$-minimal complete extension.*

A fourth semantics is defined by Dung, that does not directly rely on the notion of admissibility:

Definition 4 (Stable Semantics). *Given $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ an AF, the conflict-free set $S \subseteq \mathcal{A}$ is a* stable extension *iff $\forall a \in \mathcal{A} \setminus S$, S attacks a.*

We use $\mathsf{co}(\mathcal{F})$, $\mathsf{pr}(\mathcal{F})$, $\mathsf{gr}(\mathcal{F})$ and $\mathsf{st}(\mathcal{F})$ for the sets of (respectively) complete, preferred, grounded and stable extensions. Notice that, for any $\mathcal{F}$, $\mathsf{st}(\mathcal{F}) \subseteq \mathsf{pr}(\mathcal{F}) \subseteq \mathsf{co}(\mathcal{F})$, $\mathsf{pr}(\mathcal{F}) \neq \emptyset$, and $|\mathsf{gr}(\mathcal{F})| = 1$.

Example 2. The extensions of $\mathcal{F}$ from Example 1 are provided in the second column of Table 1.

For further details about these semantics, as well as other semantics that have been defined subsequently, we refer the reader to [2,18].

From the set of extensions of an AF, we can determine the acceptability status of an argument. Two classical reasoning modes have been defined, namely $\mathsf{Cred}_\sigma(\mathcal{F}) = \bigcup \sigma(\mathcal{F})$ (respectively $\mathsf{Skep}_\sigma(\mathcal{F}) = \bigcap \sigma(\mathcal{F})$) which denotes the set of credulously (respectively skeptically) accepted arguments of $\mathcal{F}$.

Example 3. The credulously and skeptically accepted arguments in $\mathcal{F}$ from Example 1 are given in Table 1 (third and fourth columns).

Table 1. Extensions and acceptable arguments of $\mathcal{F}$, for $\sigma \in \{\mathsf{gr}, \mathsf{st}, \mathsf{co}, \mathsf{pr}\}$.

Semantics σ	$\sigma(\mathcal{F})$	$\mathsf{Cred}_\sigma(\mathcal{F})$	$\mathsf{Skep}_\sigma(\mathcal{F})$
gr	$\{\{a\}\}$	$\{a\}$	$\{a\}$
st	$\{\{a,d,f\}\}$	$\{a,d,f\}$	$\{a,d,f\}$
co	$\{\{a,d,f\},\{a,c\},\{a\}\}$	$\{a,c,d,f\}$	$\{a\}$
pr	$\{\{a,d,f\},\{a,c\}\}$	$\{a,c,d,f\}$	$\{a\}$

The corresponding decision problems are defined by:

σ-CA Given $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $a \in \mathcal{A}$, does a belong to some σ-extension of $\mathcal{F}$?
σ-SA Given $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ and $a \in \mathcal{A}$, does a belong to each σ-extension of $\mathcal{F}$

3 Incomplete Argumentation Frameworks

This section introduces formal definitions related to Incomplete AFs, as well a complexity results and short description of SAT-based computational approaches for the main reasoning problems.

3.1 Formal Definitions

Definition 5 (Incomplete Argumentation Framework). *An incomplete argumentation framework (IAF) is a tuple* $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ *with* $\mathcal{A}, \mathcal{A}^? \subseteq \mathbf{A}$ *disjoint sets of* arguments *and* $\mathcal{R}, \mathcal{R}^? \subseteq \mathcal{A} \times \mathcal{A}$ *disjoint sets of* attacks*.*

The partition of arguments and attacks in two sets correspond to the two possible natures of elements in an incomplete AF: $\mathcal{A}$ and $\mathcal{R}$ correspond to arguments and attacks for which it is sure that they exist. On the contrary, $\mathcal{A}^?$ and $\mathcal{R}^?$ are uncertain arguments and attacks.

Example 4. In $\mathcal{I}$ from Fig. 2, $\mathcal{A} = \{a, b\}$ is the set of certain arguments, and $\mathcal{A}^? = \{c\}$ is the set of uncertain arguments. Plain arrows represent the certain attacks, *i.e.* $\mathcal{R} = \{(c, b)\}$, and dotted arrows represent the uncertain attacks, *i.e.* $\mathcal{R}^? = \{(b, a)\}$.

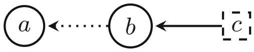

Fig. 2. An example of IAF $\mathcal{I}$

Classical reasoning methods for IAFs are based on the notion of *completion*, which are standard AFs that somehow correspond to a possible way to solve the uncertainty encoded in the IAF.

Definition 6 (Completion). *Given an IAF* $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$*, a* completion *is an AF* $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$ *such that* $\mathcal{A} \subseteq \mathcal{A}^* \subseteq \mathcal{A} \cup \mathcal{A}^?$ *and* $\mathcal{R}_{|\mathcal{A}^*} \subseteq \mathcal{R}^* \subseteq (\mathcal{R} \cup \mathcal{R}^?)_{|\mathcal{A}^*}$*.*[1]

Example 5. Continuing the previous example, we see that $\mathcal{I}$ has four completions (Fig. 3). In $\mathcal{F}_1^*$, none of the uncertain element is included, while on the contrary $\mathcal{F}_4^*$ includes all the uncertain elements. In the middle, $\mathcal{F}_2^*$ and $\mathcal{F}_3^*$ include either the argument c, or the attack (b, a).

3.2 Reasoning with IAFs

Completion-Based Reasoning. The main approach for reasoning with IAFs consists in verifying whether some property of interest (*e.g.* the credulous or skeptical acceptability of a given argument, or the fact that a given set of arguments is an extension) is true in some completion (*possible reasoning*) or in each completion (*necessary reasoning*). This means that each decision problem studied

[1] For R a set of attacks and A a set of arguments, we define the projection of R on A by $R_{|A} = R \cap (A \times A)$.

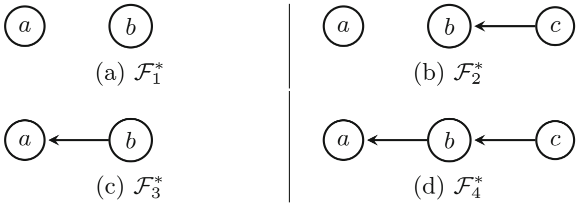

Fig. 3. The completions of $\mathcal{I}$

in the literature on AFs can be adapted in two ways when IAFs are considered. This approach was first studied for subclasses of IAFs, namely Attack-Incomplete AFs [4] (IAFs where only attacks can be uncertain, *i.e.* $\mathcal{A}^? = \emptyset$) and Argument-Incomplete AFs [7] (IAFs where only arguments can be uncertain, *i.e.* $\mathcal{R}^? = \emptyset$). These first works have been generalized to the full IAF model in [3,6].

In this paper, we focus on acceptability problems studied in [5], *i.e.*[2]

- σ-PCA Given $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ an IAF and $a \in \mathcal{A}$, is there a completion $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$ such that a is credulously accepted in $\mathcal{F}^*$ under σ?
- σ-NCA Given $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ an IAF and $a \in \mathcal{A}$, for each completion $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$, is a a credulously accepted in $\mathcal{F}^*$ under σ?
- σ-PSA Given $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ an IAF and $a \in \mathcal{A}$, is there a completion $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$ such that a is skeptically accepted in $\mathcal{F}^*$ under σ?
- σ-NSA Given $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ an IAF and $a \in \mathcal{A}$, for each completion $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$, is a a skeptically accepted in $\mathcal{F}^*$ under σ?

Example 6. Continuing the previous example, observe that a is possibly credulously accepted, as well as possible skeptically accepted, under most classical semantics. Indeed, a belongs to the single (grounded, stable, preferred, complete) extension of $\mathcal{F}_1^*$, $\mathcal{F}_2^*$ and $\mathcal{F}_4^*$, but it is not accepted in $\mathcal{F}_3^*$. No argument is necessarily accepted: a is not accepted in $\mathcal{F}_3^*$, b is not accepted in $\mathcal{F}_2^*$ and $\mathcal{F}_4^*$, and c is not accepted in $\mathcal{F}_1^*$ and $\mathcal{F}_3^*$.

Direct Reasoning. It is possible to propose reasoning methods for IAFs which do not require to consider the notion of completion. It was first proposed for so-called Partial AFs (which correspond to Attack-Incomplete AFs, and were first defined in [10] where they are used during a process of merging several AFs), the main idea is that basic notions of conflict-freeness and defense can be re-defined to take into account uncertain knowledge. These basic notions can be combined to obtain various versions of admissibility [9]. It has been proposed recently to generalize this approach to IAFs [30].

One can summarize the approach from [30] by saying that conflicts where uncertainty is involved can either be considered as serious or not. This yields two families of semantics.

[2] See [31] for an overview of other relevant decision problems.

Definition 7 (Weak Admissibility [30]). *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, a set $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ is* weakly conflict-free *if $\forall a, b \in S \cap \mathcal{A}$, $(a, b) \notin \mathcal{R}$. Then, given $a \in \mathcal{A} \cup \mathcal{A}^?$, S* weakly defends *a if $\forall b \in \mathcal{A}$ such that $(b, a) \in \mathcal{R}$, $\exists c \in S \cap \mathcal{A}$ s.t. $(c, b) \in \mathcal{R}$. Finally, S is* weakly admissible *if it is weakly conflict-free and it weakly defends all its elements.*

Weak conflict-freeness means that two arguments can be accepted together when there is an uncertain conflict between them, *i.e.* either one of the arguments involved in the conflict is uncertain, or the attack between them is uncertain. If we consider that uncertain conflicts are not serious threats in the reasoning, then there is no need to defend an argument against uncertain attacks and uncertain attackers, hence the definition of weak defense.

Definition 8 (Strong Admissibility [30]). *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, a set $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ is* strongly conflict-free *if $\forall a, b \in S$, $(a, b) \notin \mathcal{R} \cup \mathcal{R}^?$. Then, given $a \in \mathcal{A} \cup \mathcal{A}^?$, S* strongly defends *a if $\forall b \in \mathcal{A} \cup \mathcal{A}^?$ such that $(b, a) \in \mathcal{R} \cup \mathcal{R}^?$, $\exists c \in S \cap \mathcal{A}$ s.t. $(c, b) \in \mathcal{R}$. Finally, S is* strongly admissible *if it is strongly conflict-free and it strongly defends all its elements.*

Contrary to the weak version, strong admissibility assumes that all internal conflicts are bad, and all attackers must be counter-attacked (even the uncertain ones). [30] then defines the weak and strong versions of the preferred and complete semantics, and proposes also an adaptation of the stable semantics to this setting. For $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{pr}, \mathsf{st}\}$, we use respectively σ_S and σ_W to denote the strong and weak counter-parts of these semantics.

Example 7. Consider again the IAF from Fig. 2. The set of arguments $S = \{a, b, c\}$ is weakly admissible. Indeed, none of the conflicts is certain (either the attack is uncertain, namely (b, a), or the attacker is uncertain, namely c). So S is weakly conflict-free, and moreover, none of the arguments requires to be defended, so they are (trivially) weakly defended as well. Notice that this set is not strongly conflict-free. Now assume the existence of a certain argument $d \in \mathcal{A}$, such that $(d, c) \in \mathcal{R}$. This time, c is not weakly defended because it has one certain attacker which is not counter-attacked. But $S' = \{a, b, d\}$ is weakly conflict-free, and weakly admissible.

Complexity and Algorithms. Table 2 presents the complexity for the various decision problems discussed earlier. See [31] for an overview of other complexity results regarding IAFs. Given a semantics σ, σ-CA (respectively σ-SA) corresponds to credulous (respectively skeptical) acceptability for AFs, while σ_X-CA (respectively σ_X-SA) is the corresponding problem for the σ_X semantics of IAFs (where $X \in \{S, W\}$).

For $\mathcal{C}$ a complexity class of the polynomial hierarchy, $\mathcal{C}$-c means $\mathcal{C}$-complete, *i.e.* the corresponding problem is one of the hardest problem of the class $\mathcal{C}$. "Trivial" means that the answer to the question is trivially "no" for all instance. It comes from the fact that $\emptyset$ is always an admissible set, hence there is no

Table 2. Complexity of acceptability for AFs and IAFs.

σ	σ-CA	σ-SA	σ-PCA	σ-NCA	σ-PSA	σ-NSA	σ_X-CA	σ_X-SA
ad	NP-c	trivial	NP-c	Π_2^{P}-c	trivial	trivial	NP-c	trivial
st	NP-c	coNP-c	NP-c	Π_2^{P}-c	Σ_2^{P}-c	coNP-c	NP-c	coNP-c
co	NP-c	P	NP-c	Π_2^{P}-c	NP-c	coNP-c	NP-c	in coNP
gr	P	P	NP-c	coNP-c	NP-c	coNP-c	?	?
pr	NP-c	Π_2^{P}-c	NP-c	Π_2^{P}-c	Σ_3^{P}-c	Π_2^{P}-c	NP-c	Π_2^{P}-c

skeptically accepted argument with respect to ad. Finally, the question marks indicate open questions.

For the various reasoning tasks described previously, computational approaches based on SAT have been proposed, and experimental studies have shown their scalability [3,30].

4 Constrained Incomplete Argumentation Frameworks

4.1 The Disjunction Problem

An important question with reasoning formalisms is "What can be represented with this formalism?". In the case of abstract argumentation, this question has arisen in a context of belief revision and belief merging, adapted for argumentation frameworks [11,14]. These works propose two step processes to revise or merge argumentation frameworks: first revise (or merge) the extensions, using an adaptation of propositional belief revision (or merging) operators [23,24]. Then, from the revised (or merged) extensions, generate a set of argumentation frameworks that correspond to these extensions. Indeed, it is necessary to use a set instead of a single AF, because it is known that some sets of extensions cannot be represented by a single AF [19]. However, from a purely logical point of view, it is not surprising: the result of a revision (or merging) can be a disjunction of several (equally plausible) pieces of information. In the case of AFs, if the result of the first step is $\{\{a\}, \{a, b\}\}$, then it makes sense to have two AFs, one where a attacks b, and one where this attack does not exist. This set of AFs can be seen as a "disjunction" of its elements. We have thus been interested in the question whether IAFs would be a suitable formalism for representing any set of extensions, or any set of AFs, *i.e.* given $\mathfrak{F}$ a set of AFs, is there an IAF $\mathcal{I}$ such that $\mathtt{comp}(\mathcal{I}) = \mathfrak{F}$. The answer to this question is negative, as shown in this example (borrowed from [29]).

Example 8. Suppose that the result of revising an AF is the set $\mathfrak{F} = \{\mathcal{F}_1 = \langle\{a, b\}, \{(b, a)\}\rangle, \mathcal{F}_2 = \langle\{a, c\}, \{(c, a)\}\rangle\}$. The question is to determine whether this set can be compactly represented by a single IAF. Towards a contradiction, suppose that there is an IAF $\mathcal{I} = \langle\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?\rangle$ s.t. $\mathtt{comp}(\mathcal{I}) = \mathfrak{F}$. Since a belongs to both $\mathcal{F}_1$ and $\mathcal{F}_2$, it must belong to the certain arguments $\mathcal{A}$. On the contrary, the uncertain arguments are $\mathcal{A}^? = \{b, c\}$, each of them belongs to some (but not all) completions. $\mathcal{A} = \{a\}$ and $\mathcal{A}^? = \{b, c\}$ imply the existence of some completions that only contain a, and some completions that contain the three arguments a, b, c. This is not the case in $\mathfrak{F}$. So $\mathcal{I}$ does not exist.

4.2 Towards Higher Expressiveness: Rich IAFs

Control Argumentation Frameworks (CAFs) [15,28] introduce several novelties to IAFs. One of them is a new kind of uncertain information, namely conflicts with uncertain direction. In [27], we borrow this new kind of uncertainty and add it to IAFs, thus defining Rich IAFs (RIAFs).

Definition 9. *A* Rich Incomplete Argumentation Framework *(RIAF) is a tuple $r\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \leftrightarrow^? \rangle$, where $\mathcal{A}$ and $\mathcal{A}^?$ are disjoint sets of arguments, and $\mathcal{R}, \mathcal{R}^?, \leftrightarrow^? \subseteq (\mathcal{A} \cup \mathcal{A}^?) \times (\mathcal{A} \cup \mathcal{A}^?)$ are disjoint sets of attacks, such that $\leftrightarrow^?$ is symmetric.*

The new relation $\leftrightarrow^?$ borrowed from CAFs [15] is a symmetric (uncertain) conflict relation: if $(a, b) \in \leftrightarrow^?$, then we are sure that there is a conflict between a and b, but not of the direction of the attack. This new relation impacts the definition of completions.

Definition 10 (Completion of a RIAF). *Given $r\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \leftrightarrow^? \rangle$, a completion of $r\mathcal{I}$ is $\mathcal{F}^* = \langle \mathcal{A}^*, \mathcal{R}^* \rangle$, such that*

- $\mathcal{A} \subseteq \mathcal{A}^* \subseteq \mathcal{A} \cup \mathcal{A}^?$*;*
- $\mathcal{R}_{|\mathcal{A}^*} \subseteq \mathcal{R}^* \subseteq \mathcal{R}_{|\mathcal{A}^*} \cup \mathcal{R}^?_{|\mathcal{A}^*} \cup \leftrightarrow^?_{|\mathcal{A}^*}$*;*
- *if $(a, b) \in \leftrightarrow^?_{|\mathcal{A}^*}$, then $(a, b) \in \mathcal{R}^*$ or $(b, a) \in \mathcal{R}^*$ (or both).*

Example 9. Assume the RIAF $r\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \leftrightarrow^? \rangle$, described by Fig. 4. Its completions are shown at Fig. 5.

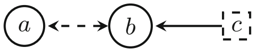

Fig. 4. The RIAF $r\mathcal{I}$

In [27] we also proved that RIAFs are strictly more expressive than IAFs, in the sense that there are sets of AFs that can be the completions of a RIAF, but not of an IAF.

Proposition 1 (Relative Expressivity of IAFs and RIAFs). *RIAFs are strictly more expressive than IAFs,* i.e.

- *for any IAF $\mathcal{I}$, there exists a RIAF $r\mathcal{I}$ such that* $\texttt{comp}(\mathcal{I}) = \texttt{comp}(r\mathcal{I})$*;*
- *there exists a RIAF $r\mathcal{I}$ such that there is no IAF $\mathcal{I}$ with* $\texttt{comp}(\mathcal{I}) = \texttt{comp}(r\mathcal{I})$.

However, one can also prove that some sets of AFs cannot be represented by a RIAF. Indeed, the additional expressiveness of RIAFs does not solve the problem illustrated by Example 8.

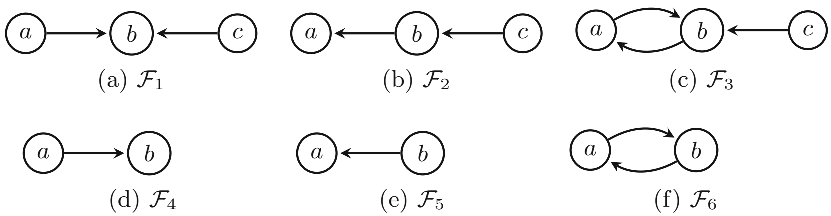

Fig. 5. The completions of $r\mathcal{I}$

4.3 Constrained IAFs

To improve the expressiveness of the formalism, instead of adding different types of attacks, we add a propositional formula which serves as a constraint over the set of completions. The idea is to restrict the set of completions of the IAF which can be used for possible and necessary reasoning.

Definition 11 (Constraint). *Given $\mathcal{A}$ a set of arguments, we define the set of propositional atoms $Prop_{\mathcal{A}} = Arg_{\mathcal{A}} \cup Att_{\mathcal{A}}$ where $Arg_{\mathcal{A}} = \{\mathrm{arg}_a \mid a \in \mathcal{A}\}$ and $Att_{\mathcal{A}} = \{\mathrm{att}_{a,b} \mid (a,b) \in \mathcal{A} \times \mathcal{A}\}$. Then, $\mathcal{L}_{\mathcal{A}}$ is the propositional language built from $Prop_{\mathcal{A}}$ with classical connectives $\{\neg, \vee, \wedge\}$.*

The satisfaction of a constraint by an AF is defined as follows.

Definition 12 (Constraint Satisfaction). *Given $\mathcal{A}$ a set of arguments, and $\phi \in \mathcal{L}_{\mathcal{A}}$ a formula, the set of models of ϕ is denoted* $\mathrm{mod}(\phi)$. *An AF $\mathcal{F} = \langle \mathcal{A}', \mathcal{R} \rangle$ with $\mathcal{A}' \subseteq \mathcal{A}$ and $\mathcal{R} \subseteq \mathcal{A}' \times \mathcal{A}'$* satisfies *$\phi$ iff there is a model $\omega \in \mathrm{mod}(\phi)$ s.t. $\mathcal{A}' = \{a \in \mathcal{A} \mid \omega(\mathrm{arg}_a) = \top\}$, and $\mathcal{R} = \{(a,b) \in \mathcal{A} \times \mathcal{A} \mid \omega(\mathrm{att}_{a,b}) = \top)\}$.*

Definition 13 (Constrained IAF). *A* Constrained Incomplete Argumentation Framework *(CIAF) is a tuple $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$, where $\langle \mathcal{A}, \mathcal{A}^?, \mathcal{A}, \mathcal{A}^? \rangle$ is an IAF, and $\phi \in \mathcal{L}_{\mathcal{A} \cup \mathcal{A}^?}$ is a constraint.*

The constraint ϕ is used to select a subset of the completions of the IAF $\mathcal{I}_{c\mathcal{I}} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$. The completions of a CIAF are then defined as follows.

Definition 14 (Completions of a CIAF). *Given $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ a CIAF, we define its set of completions by $\texttt{comp}(c\mathcal{I}) = \{c \in \texttt{comp}(\mathcal{I}_{c\mathcal{I}}) \mid c$ satisfies $\phi\}$ where $\mathcal{I}_{c\mathcal{I}} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$.*

Example 10. Let $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ be a CIAF s.t. $\mathcal{I}_{c\mathcal{I}} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ is the IAF from Fig. 6, and $\phi = \mathrm{att}_{e,a} \wedge \mathrm{arg}_f$. The completions of $\mathcal{I}_{c\mathcal{I}}$ are given in Fig. 7. Only two of them satisfy ϕ, namely $\mathcal{F}_5$ (Fig. 7c) and $\mathcal{F}_6$ (Fig. 7f). So $\texttt{comp}(c\mathcal{I}) = \{\mathcal{F}_5, \mathcal{F}_6\}$. This means, for instance, that f is necessary skeptically accepted with respect to $c\mathcal{I}$, while it is not with respect to $\mathcal{I}_{c\mathcal{I}}$.

Now we recall the result from [29] about the expressiveness of CIAFs. This result is based on Definition 15, which introduces a constraint satisfied by only one AF.

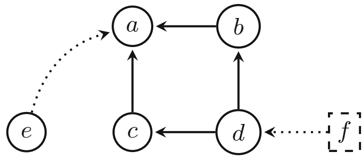

Fig. 6. The IAF $\mathcal{I}$

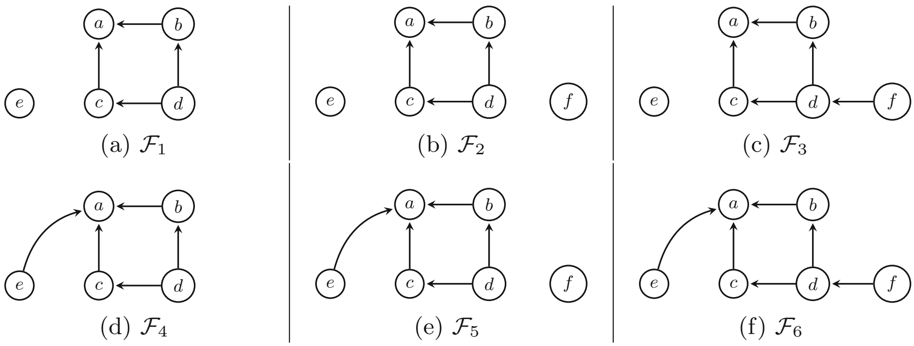

Fig. 7. The completions of $\mathcal{I}$

Definition 15. *Given $\mathcal{A}$ a set of arguments, and $\mathcal{F} = \langle \mathcal{A}', \mathcal{R} \rangle$ with $\mathcal{A}' \subseteq \mathcal{A}$, and $\mathcal{R} \subseteq \mathcal{A}' \times \mathcal{A}'$, we define $\psi_{\mathcal{F}} \in \mathcal{L}_{\mathcal{A}}$ as*

$$\psi_{\mathcal{F}} = (\bigwedge_{a \in \mathcal{A}'} \text{arg}_a) \wedge (\bigwedge_{a \in \mathcal{A} \setminus \mathcal{A}'} \neg \text{arg}_a) \wedge (\bigwedge_{(a,b) \in \mathcal{R}} \text{att}_{a,b}) \wedge (\bigwedge_{(a,b) \in (\mathcal{A} \times \mathcal{A}) \setminus \mathcal{R}} \neg \text{att}_{a,b})$$

Proposition 2. *Let $\mathfrak{F} = \{\mathcal{F}_1 = \langle \mathcal{A}_1, \mathcal{R}_1 \rangle, \dots, \mathcal{F}_n = \langle \mathcal{A}_n, \mathcal{R}_n \rangle\}$ be a set of AFs. There is a CIAF $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ s.t. $\texttt{comp}(c\mathcal{I}) = \mathfrak{F}$.*

Intuitively, a simple CIAF that does the job consists of all the arguments and attacks from $\mathfrak{F}$ defined as uncertain, and then ϕ is the disjunction of the $\psi_{\mathcal{F}}$ formulas, for $\mathcal{F} \in \mathfrak{F}$. Let us prove this result with a simple illustration of a well suited CIAF.

Proof. Let us build a CIAF $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ s.t. $\texttt{comp}(c\mathcal{I}) = \mathfrak{F}$. To do that, we first choose $\mathcal{A} = \emptyset$ and $\mathcal{A}^? = \bigcup_{i=1}^n \mathcal{A}_i$, *i.e.* all the arguments that appear in an AF from $\mathfrak{F}$ are uncertain. Similarly, all the attacks are uncertain, *i.e.* $\mathcal{R} = \emptyset$ and $\mathcal{R}^? = \bigcup_{i=1}^n \mathcal{R}_i$. With all these choices, we define an IAF that has all the possible completions on arguments and attacks from $\mathfrak{F}$. In order to restrict the completions to exactly the AFs in $\mathfrak{F}$, we define $\phi = \bigvee_{i=1}^n \psi_{\mathcal{F}_i}$, where $\psi_{\mathcal{F}_i}$ is the formula that is only satisfied by the AF $\mathcal{F}_i$, following Definition 15. The AFs that satisfy ϕ are exactly the ones in $\mathfrak{F}$, so we have $\texttt{comp}(c\mathcal{I}) = \mathfrak{F}$.

A consequence of this result is that any set of extensions can be represented by an IAF.

Proposition 3. *Let $\mathfrak{E} = \{E_1, \dots, E_n\}$ be a set of non-empty extensions, and $\sigma \in \{\mathsf{co}, \mathsf{pr}, \mathsf{st}, \mathsf{gr}\}$. There is a CIAF $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ s.t. $\bigcup_{c \in \mathtt{comp}(c\mathcal{I})} \sigma(c) = \mathfrak{E}$.*

Proof. First, let us define $\mathcal{A} = \bigcup_{i=1}^{n} E_i$, *i.e.* it is the set of all the arguments that appear in some extension. Then, for each $E_i \in \mathfrak{E}$, we define $\mathcal{F}_i = \langle \mathcal{A}, \mathcal{R}_i \rangle$ s.t. $\mathcal{R}_i = \{(a, b) \mid a \in E_I, b \in \mathcal{A} \setminus E_i\}$, *i.e.* each argument in E_i is unattacked, and it attacks all the arguments that are not in the extension. For any σ defined in this paper,[3] E_i is the only extension of $\mathcal{F}_i$. Thus, $\bigcup_{i=1}^{n} \sigma(\mathcal{F}_i) = \mathfrak{E}$. From Proposition 2, there is $c\mathcal{I}$ s.t. $\mathtt{comp}(c\mathcal{I}) = \{\mathcal{F}_1, \dots, \mathcal{F}_n\}$. This concludes the proof.

Corollary 1. *Let $\mathfrak{E} = \{E_1, \dots, E_n\}$ be a set of extensions, and $\sigma \in \{\mathsf{co}, \mathsf{pr}, \mathsf{gr}\}$. There is a CIAF $c\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?, \phi \rangle$ s.t. $\bigcup_{c \in \mathtt{comp}(c\mathcal{I})} \sigma(c) = \mathfrak{E}$.*

Proof. For non-empty extensions, Proposition 3 can be applied. If some $E_i \in \mathfrak{E}$ is empty, then simply build $\mathcal{F}_i = \langle \mathcal{A}, \mathcal{R}_i \rangle$ where $\mathcal{R}_i = \{(a, a) \mid a \in \mathcal{A}\}$, *i.e.* each argument is self-attacking. The unique σ-extension of $\mathcal{F}_i$ is $E_i = \emptyset$.

From Propositions 2 and 3 and Corollary 1, we deduce that the result of any revision or merging operator from [11,14] can be represented by a CIAF, or said otherwise any set of extensions is realizable [19] when CIAFs are used as the knowledge representation formalism instead of AFs. The same result has been demonstrated independently in [22].

Besides the interest of CIAFs for representing the result of revision or merging operators, more generally they allow to represent the epistemic state of any agent about the current and future states of a debate. Assume than agent A is debating with agent B regarding two arguments a and b which are mutually exclusive. Agent A knows that agent B has some preferences over these arguments, but she does not know exactly agent B's preferences. This means that in agent B's state of mind, either a attacks b, or b attacks a, but not both. These two possible AFs corresponding to agent A's knowledge about B cannot be encoded into a single (R)IAF, but the result described in this section show that they can be represented by a single CIAF.

4.4 Complexity

In the previous section, we have shown that RIAFs are strictly more expressive than IAFs, and CIAFs even more, since any set of AFs (or extensions) can be represented by a CIAF. However, this expressiveness does not come at the price of an increased complexity, compared to the complexity of standard IAFs. More precisely, [29] has shown that the complexity of the decision problems PCA and NSA are the same as in the case of IAFs. The complexity of other decision problems for CIAFs remains an open question.

[3] And arguably most semantics defined in the literature.

5 Related Work

As mentioned previously, Control Argumentation Frameworks [15,28] are a generalization of IAFs (or more precisely, of RIAFs). The additional component, namely the *control part* of the CAF, represents arguments and attacks that can be used by an agent to influence the outcome of the argumentation process. Complexity and algorithms were provided in [15,28,33], and an application of this framework to automated negotiation was proposed in [16,17]. CAFs are intrinsically made for strategic applications of argumentation (and in this sense, they are more general than IAFs), but they do not have the maximal expressiveness of CIAFs regarding their set of completions. For this reason, combining CIAFs and CAFs is an interesting future work.

In IAFs (and the related frameworks discussed in this paper), uncertainty is purely qualitative, in the sense that the agent knows that some argument or attack may exist or not, but without a possibility to quantify how plausible is the existence of this element. Probabilistic Argumentation Frameworks (PrAFs) [25] can be seen as an enriched version of IAFs, where the existence of each element is associated with a probability. In the case where such a probability is available, it allows to have more precise inference, for instance because some completions are more probable than other ones. A probabilistic version of CAFs has also been defined [21]. The relation between PrAFs and IAFs has been discussed in [3].

6 Conclusion

There are interesting research tracks regarding CIAFs. In particular, the method described in this paper to exhibit a CIAF corresponding to a set of AFs (or extensions) only works for proving the existence of this CIAF, but it may not be suited to real application of this formalism. For instance, in the case of belief revision or merging, a classical principle is minimal change: we expect the result to be as close as possible to the initial knowledge. To ensure that the graph structure of the CIAF (*i.e.* the sets $\mathcal{A}$, $\mathcal{A}^?$, $\mathcal{R}$ and $\mathcal{R}^?$) is as close as possible to the input graph, one can use techniques similar to distance minimization used in the literature [10,12]. Regarding the constraint, there are two aspects. The first one applies in the case where the initial knowledge of the agent is a CIAF, and not simply an AF. In this case, one can expect that the constraint in the revised CIAF is close to the constraint in the initial CIAF. Then, we can see that the formula defined in the proof of Proposition 2 can be exponentially large in the worst case. To avoid this kind of issue, one can apply techniques from knowledge compilation, in order to obtain an equivalent formula that would be more succinct [13]. Among other future works, one can mention the combination of CIAFs with CAFs or PrAFs, which would allows better representation of the epistemic states of agents participating in a negotiation [16,17,21].

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