Joint Spectrum Access and Pricing in Cognitive Radio Networks with Elastic Traffic

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***Abstract*—This paper studies the economic interactions between Secondary Users and Primary Operators in a Cognitive Radio Network scenario. Secondary Users transmit their traffic, eventually splitting it over multiple available frequency spectra, each owned by an independent primary network operator. Users are charged a fixed price per unit of bandwidth used, and face spectrum access costs. The transmission rate of each secondary user is assumed to be function of network congestion (like for TCP traffic) and the price per bandwidth unit. Primary operators sell spare bandwidth to secondary users, and set spectrum access prices to maximize their revenue.**

We provide sufficient conditions for the existence and uniqueness of the Nash equilibrium considering a peculiar class of spectrum pricing functions, viz. polynomial functions, which lead to efficient spectrum allocation, and we derive optimal price and spectrum allocation settings. Finally, we discuss numerical cognitive radio network examples that provide insights into the model's solution.

***Index Terms*: - Cognitive Radio Networks, Spectrum Access, Pricing, Stackelberg Game, Elastic Traffic.**

I. Introduction

Cognitive Radio Networks (CRNs) can provide high bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques. The inefficient usage of the available spectrum [1] can be improved through opportunistic access to the licensed bands without interfering with the existing users [2].

In a CRN, a *primary* (or licensed) user has a license to operate in a certain spectrum band; his access is generally controlled by the Primary Operator (PO) and should not be affected by the operations of any other unlicensed user. On the other hand, unlicensed (*secondary*) users have no spectrum license, and they implement additional functionalities to share the licensed spectrum band without interfering with primary users.

This work focuses on the joint dynamic spectrum access and pricing problems in cognitive radio networks using Game Theory. We consider multiple primary operators, which are owners of spectrum bands and set prices for their licensed spectrum, and a set of secondary users, which compete in a non-cooperative way for the spectrum bands left available by primary users. As a consequence, Game Theory is the natural framework to study the interactions among such users.

Non-cooperative games for competitive spectrum access in cognitive radio networks have been recently considered in [2], [3], [4], [5], [6], [7], [8]. The works in [7], [8], which propose static and dynamic spectrum sharing schemes as well as spectrum pricing techniques, are related to our work, but they present the following limitations: (1) only one primary operator is considered, while it would be more realistic to take into account different primary operators that provide spectrum access opportunities to the secondary users, and (2) infinitesimally small secondary users are assumed; secondary users form a secondary service, which is represented by a demand function. However, it is more interesting to study a more general scenario, where multiple primary operators coexist and provide spectrum access to a *finite* set of secondary users characterized by elastic traffic demands.

This paper overcomes these limitations by proposing a novel game theoretic model that solves the joint problem of spectrum allocation and price setting, considering both multiple Primary Operators (POs), which set prices for their licensed spectrum, and a finite set of Secondary Users (SUs), which are characterized by elastic traffic demands that can be transmitted over one or multiple frequency spectra.

We model the interaction between POs and SUs as a Stackelberg game [9]: POs set their prices and SUs respond by presenting a certain amount of flow to the network. Secondary users do not cooperate among themselves, thus leading to a Nash game. We study analytically the existence and uniqueness of the Nash Equilibrium Point (NEP) for our Stackelberg game. Next, we obtain explicit expressions for the secondary users' flows and the optimal prices, along with the conditions under which the NEP exits and is unique.

Finally, we analyze and discuss several numerical examples that illustrate the following results: (1) the optimal price that primary operators should impose is independent of the number of secondary users, and it is proportional to the average utility per traffic unit experienced by such users; (2) as the number of secondary users grows, the revenue achieved by primary operators increases until it reaches a saturation point.

The paper is structured as follows: Section II introduces the cognitive radio network model, including POs' and SUs' objective functions, as well as the considered cost functions. Section III demonstrates the existence and uniqueness of the Nash Equilibrium Point, while Section IV computes the NEP along with the conditions under which such equilibrium exists and is unique. Section V determines the NEP for the special case of symmetrical secondary users. Section VI discusses numerical examples that illustrate how our model captures interesting aspects of the interaction between primary operators and secondary users. Finally, Section VII concludes this paper.

II. Network Model

The adopted model is a cognitive radio wireless system with a set $\mathcal{N} = \{1, \dots N\}$ of Primary Operators (POs), each operating on a separate frequency spectrum, F_n, and a set $\mathcal{U} = \{1, \dots I\}$ of Secondary Users (SUs), willing to utilize the frequency spectra $\{F_1, \dots F_N\}$.

Each Secondary User can transmit simultaneously over multiple spectrum bands, splitting his traffic over the set of available channels, thus choosing which primary operators will transport his traffic.

Users demands are *elastic* (like for TCP traffic), in the sense that they are function of the prices set by POs and the costs due to link congestion, as well as of the utility perceived in transmitting the traffic over the available channels.

Let f_l^i denote the expected flow that SU i sends on wireless channel l. The secondary user flow configuration $f^i = \{f_1^i, \dots, f_N^i\}$ is called a spectrum access *strategy* of SU i, and the set of strategies $H^i = \{f^i \in R^N : f_l^i \geq 0, l \in \mathcal{N}\}$ is called the spectrum access strategy space of SU i.

The system flow configuration $f = \{f^1, \dots, f^I\}$ is called a spectrum access *strategy profile*, and takes values in the product strategy space H. Furthermore, let f^{-i} represent the flow configuration of all users except SU i.

Each secondary user $i \in \mathcal{U}$ maximizes his degree of satisfaction (his objective function $F^{u,i}$, defined in the following), which we assume has a component related to the throughput (*utility*) and another one related to costs (*disutility*).

On the other hand, each Primary Operator behaves selfishly, and plays to maximize its own profit. Each PO l defines its own *price strategy* $p_l \in R$, where p_l represents the price per bandwidth unit charged by PO l for all the traffic that passes through wireless channel F_l. The collection of the PO strategies builds up the price strategy profile $p = (p_1, \dots p_N) \in R^N$. Let p_{-l} represent the price strategy of all POs except PO l.

The vector $(f, p) \in R^{IN+N}$ grouping both the spectrum access and price strategy profiles is referred to as network strategy profile, and represents the solution of the game where both SUs and POs operate simultaneously.

A. Secondary User Objective Function

We associate to SU $i \in \mathcal{U}$ the objective function $F^{u,i}$, which is a function of the flow transmitted on each wireless channel as well as of the prices set by Primary Operators:

$$F^{u,i}(f^i, f^{-i}, p) = Q^i(f^i) - [\sum_{l \in \mathcal{N}} f_l^i \cdot J_l^i(f^i, f^{-i}) + + \sum_{l \in \mathcal{N}} f_l^i \cdot p_l(f_l^i, f_l^{-i})]. \quad (1)$$

The first term, $Q^i(f^i)$, represents the utility for transmitting a total amount of flow $\sum_{l \in \mathcal{N}} f_l^i = f^i$. The second term encompasses two components: the first one, $\sum_{l \in \mathcal{N}} f_l^i \cdot J_l^i(f^i, f^{-i})$, is the disutility due to link delay and the second one, $\sum_{l \in \mathcal{N}} f_l^i \cdot p_l(f_l^i, f_l^{-i})$ corresponds to the price paid by secondary user i to the Primary Operators. Recall that $p_l(f_l^i, f_l^{-i})$ is the price per bandwidth unit set by the $l-$th PO.

In the following we assume that $Q^i(f^i)$ is equal to $\sum_{l \in \mathcal{N}} \alpha_l^i \cdot f_l^i$ and $p_l(f_l^i, f_l^{-i})$ is constant and equal to p_l; α_l^i therefore represents the utility of SU i per unit of transmitted flow, on wireless channel l. Hence, the objective function of SU i becomes as follows:

$$F^{u,i}(f^i, f^{-i}, p) = \sum_{l \in \mathcal{N}} \alpha_l^i \cdot f_l^i - \sum_{l \in \mathcal{N}} f_l^i \cdot [J_l^i(f^i, f^{-i}) + p_l]. \quad (2)$$

Each SU i maximizes his objective function $F^{u,i}$ over all his flow configurations:

$$\max_{f_l^i} \left\{ \sum_{l \in \mathcal{N}} \alpha_l^i \cdot f_l^i - \sum_{l \in \mathcal{N}} f_l^i \cdot [J_l^i(f^i, f^{-i*}) + p_l^*] \right\}, \quad (3)$$

where p_l^* is the optimal price set by Primary Operator l and f^{-i*} are the optimal flows of all secondary users $j \in \mathcal{U}$, with $j \neq i$.

B. Primary Operator Utility Function

The utility function of the $l-$th Primary Operator, U_l^{PO}, is given by $U_l^{PO}(f, p_l, p_{-l}) = p_l \cdot f_l$, where $f_l = \sum_{i \in \mathcal{U}} f_l^i$ is the total amount of flow on wireless channel l.

Each SP l maximizes its utility U_l^{PO} over all its price strategies: $\max_{p_l}\{p_l \cdot f_l\}$.

C. Polynomial Link Costs

Following the guidelines of [10], this work considers a class of polynomial link cost functions originally adopted in the context of road traffic modeling [11]. Such costs have appealing properties that lead to predictable and efficient network flows, ensuring the uniqueness of the Nash equilibrium point.

More specifically, we assume that each wireless channel l is characterized by the following cost function:

$$cost(l) = a_l \cdot (f_l)^{\beta(l)} + b_l, \forall l \in \mathcal{N}, \quad (4)$$

where a_l, b_l and $\beta(l)$ are channel-specific positive parameters, and f_l, as defined above, is the total amount of flow that is transmitted over wireless channel l.

This is the cost adopted by the US Bureau of Public Roads [11]. The additive term b_l here can be interpreted as an additional fixed toll per traffic unit for the use of wireless channel l.

III. Existence and Uniqueness of the Nash Equilibrium Point

We now demonstrate the existence and uniqueness of the Nash Equilibrium Point (NEP), considering the polynomial cost function (4) defined above.

Let us denote by $F_l^{u,i}(f^i, f^{-i}, p)$ the secondary user objective function on wireless channel l, which is equal to:

$$F_l^{u,i}(f^i, f^{-i}, p) = \alpha_l^i \cdot f_l^i - (f_l^i \cdot [a_l \cdot (f_l)^{\beta(l)} + b_l + p_l]) \quad (5)$$

We have therefore the following objective function for each SU $i \in \mathcal{U}$:

$$F^{u,i}(f^i, f^{-i}, p) = \sum_{l\in\mathcal{N}} \alpha_l^i \cdot f_l^i - \sum_{l\in\mathcal{N}} f_l^i \cdot [a_l \cdot (f_l)^{\beta(l)} + b_l + p_l]. \quad (6)$$

The i-th secondary user objective function (6) is continuous in $f = \{f^1, \ldots, f^I\}$ and concave in f_l^i: the second partial derivatives of $F^{u,i}(f)$ with respect to f_l^i are equal to $-a_l \cdot \beta(l) \cdot (f_l)^{[\beta(l)-2]} \cdot [2f_l + (\beta(l)-1) \cdot f_l^i]$ and are therefore negative $\forall f_l^i \geq 0$, $f_l \geq 0$ and $\beta(l) > 0$. These properties ensure the existence of the Nash equilibrium [12].

Having settled the question of existence of a NEP, it can be shown that the NEP is indeed unique under appropriate conditions [10]. The demonstration is not reported due to space constraints.

IV. Computing the Solution

This section is dedicated to computing the Nash equilibrium point. For a given price vector $p = (p_1, p_2, ..., p_N) \in R^N$, we have a non-cooperative spectrum access game between secondary users. Each SU i maximizes his objective function $F^{u,i}$, which has the following expression, considering the cost function illustrated in Section II-C:

$$F^{u,i}(f^i, f^{-i}, p) = \sum_{l\in\mathcal{N}} \alpha_l^i \cdot f_l^i - \sum_{l\in\mathcal{N}} f_l^i \cdot [a_l \cdot (f_l)^{\beta(l)} + b_l + p_l].$$

The optimal flows of the users can be obtained by solving the set of first-order conditions: for $i \in \mathcal{U}$,

$$(\partial/\partial f_l^i) F^{u,i} \begin{cases} = 0 & \text{, if } f_l^i > 0 \\ < 0 & \text{, if } f_l^i = 0 \end{cases}$$

which yields the following expression of the $i-$th secondary user flow on wireless channel l:

$$f_l^i(p_l) = \frac{\alpha_l^i - [a_l(f_l)^{\beta(l)} + b_l + p_l]}{a_l \beta(l)(f_l)^{[\beta(l)-1]}}. \quad (7)$$

If we sum over all $i \in \mathcal{U}$ we obtain:

$$f_l(p_l) = \left[\frac{\sum_{i\in\mathcal{U}} \alpha_l^i - I \cdot (b_l + p_l)}{(I + \beta(l))a_l}\right]^{1/\beta(l)}, \quad (8)$$

and the condition $f_l > 0$ implies that $p_l < \hat{p} = \sum_{i\in\mathcal{U}} \frac{\alpha_l^i}{I} - b_l$. At this stage, Primary Operator l should solve the problem of maximizing the objective function $p_l \cdot f_l(p_l) = p_l \cdot \left[\frac{\sum_{i\in\mathcal{U}} \alpha_l^i - I\cdot(b_l+p_l)}{(I+\beta(l))a_l}\right]^{1/\beta(l)}$ with respect to p_l.

The optimal value of p_l is:

$$p_l^* = \frac{\beta(l)}{1+\beta(l)} \cdot [\sum_{i\in\mathcal{U}} \alpha_l^i / I - b_l] \quad (9)$$

and the value of the optimal flow f_l^* is:

$$f_l^* = \left[\frac{\sum_{i\in\mathcal{U}} \alpha_l^i - I \cdot b_l}{a_l \cdot (1+\beta(l)) \cdot (I+\beta(l))}\right]^{1/\beta(l)} \quad (10)$$

provided that $\sum_{i\in\mathcal{U}} \alpha_l^i / I > b_l$, which ensures that $f_l^* > 0$ and $p_l^* > 0$. It is also easy to check that $p_l^* < \hat{p}$.
The optimal values of f_l^i are:

$$f_l^{i*} = \frac{\mathrm{A}}{\mathrm{D}} \quad (11)$$

where A and D have the following expressions:

$$\mathrm{A} = \alpha_l^i I(1+\beta(l))(I+\beta(l)) - \\ - \left(\sum_{i\in\mathcal{U}} \alpha_l^i\right)[I + I\beta(l) + \beta(l)^2] - I\beta(l)b_l$$

$$\mathrm{D} = I\beta(l)[a_l(1+\beta(l))(I+\beta(l))]^{1/\beta(l)} \cdot \\ \cdot \left[\sum_{i\in\mathcal{U}} \alpha_l^i - Ib_l\right]^{(1-1/\beta(l))}.$$

Now we must derive the conditions under which the individual flows are positive. It is easy to check that the denominator is positive provided that $\sum_{i\in\mathcal{U}} \alpha_l^i / I > b_l$. So $f_l^{i*} > 0 \Rightarrow [I + I\beta(l) + \beta(l)^2](\alpha_l^i - \sum_{i\in\mathcal{U}} \alpha_l^i / I) + \beta(l)\alpha_l^i > \beta(l)b_l$.
For $\beta(l) = 1$,

$$f_l^{i*} = \frac{1}{a_l} \cdot \left[\alpha_l^i - \frac{(1+2I)}{2I(1+I)} \cdot \left(\sum_{i\in\mathcal{U}} \alpha_l^i\right) - \frac{b_l}{2(1+I)}\right], \quad (12)$$

and $f_l^{i*} > 0 \Rightarrow \alpha_l^i > \frac{(1+2I)\cdot\left(\sum_{i\in\mathcal{U}} \alpha_l^i / I\right) + b_l}{2(I+1)}$.
Note that, for $\beta(l) = 1$, the condition $\sum_{i\in\mathcal{U}} \alpha_l^i / I > b_l$ implies that $\frac{(1+2I)\cdot\left(\sum_{i\in\mathcal{U}} \alpha_l^i / I\right) + b_l}{2(I+1)} > b_l$, and from the previous expression ($\alpha_l^i > \frac{(1+2I)\cdot\left(\sum_{i\in\mathcal{U}} \alpha_l^i / I\right) + b_l}{2(I+1)}$), we can write the following: $\alpha_l^i > \frac{(1+2I)\cdot\left(\sum_{i\in\mathcal{U}} \alpha_l^i / I\right) + b_l}{2(I+1)} > b_l$.

V. Solution for a special case: Symmetrical Users

We consider here a special case of the problem formulated above, where all secondary users have the same utility per unit of flow on wireless channel l, i.e., $\alpha_l^i = \alpha_l, \forall i \in \mathcal{U}, l \in \mathcal{N}$, and as a consequence they share the same objective function.

This special case permits to derive simpler equilibrium expressions, providing further insights into the game equilibria.

Proposition 1: In the considered cognitive radio wireless system with symmetrical users, the price and flow values at the Nash Equilibrium Point are given by the following expressions:

$$p_l^* = \frac{\beta(l)}{1+\beta(l)} \cdot (\alpha_l - b_l) \quad (13)$$

$$f_l^{i*} = \frac{f_l^*}{I}, \quad (14)$$

where $f_l^* = \left[\frac{I(\alpha_l - b_l)}{a_l(1+\beta(l))(I+\beta(l))}\right]^{1/\beta(l)}$.

Furthermore, for $\beta(l) = 1$, p_l^* and f_l^* become:

$$p_l^* = \frac{\alpha_l - b_l}{2} \tag{15}$$

and

$$f_l^* = \frac{I(\alpha_l - b_l)}{2a_l(I+1)}. \tag{16}$$

Proof: The above expressions of the equilibrium prices and flows can be easily derived following the same procedure used in Section IV for computing the general solution.

□

VI. Numerical Results

This section analyzes and discusses the numerical results obtained by solving our proposed game, testing the sensitivity of the achieved network equilibria to different parameters, namely the SUs' per-bandwidth utility (α_l^i) and the number of Secondary Users (I).

A. *Effect of the Utility (α_l^i)*

We first consider a Cognitive Radio Network scenario with 2 identical SUs that share a single frequency spectrum F_l (i.e., $N = 1$), owned by a PO. The parameter values are set as follows: $\alpha_l^i = 1, \forall i \in \mathcal{U}, l \in \mathcal{N}$, $a_l = 1$, $b_l = 0.5$ and $\beta(l) = 1, \forall l \in \mathcal{N}$.

The Nash Equilibrium solution is in this case $p_l^* = 1/4$ and $f_l^{1*} = f_l^{2*} = 1/12$. Figure 1 illustrates the secondary user objective function (which is identical for both users) $F_l^{u,i} = \alpha_l^i \cdot f_l^i - f_l^i \cdot [a_l \cdot (f_l)^{\beta(l)} + b_l + p_l]$ as a function of the users' flow values (f_l^i). In this Figure, the price p_l was set to $1/4$, so that the Primary Operator maximizes its revenue.

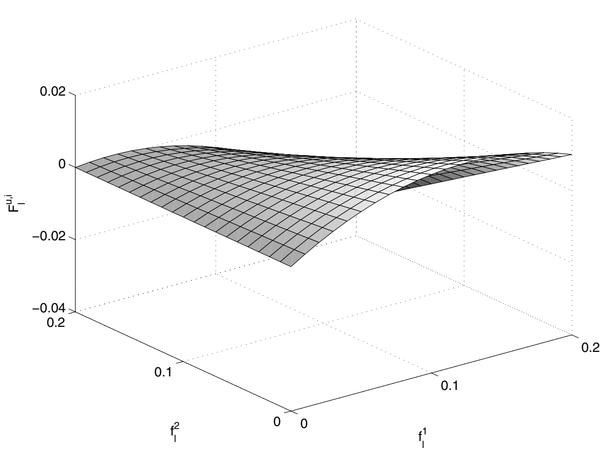

Fig. 1. Secondary user objective function in the two-user network scenario as a function of f_l^1 and f_l^2, with $p_l = p_l^* = 1/4$.

We then measured the amount of flow transmitted into the network by SUs having different α_l^i values, i.e., different per bandwidth unit utilities. To this aim, we considered a CRN scenario with $N = 2$ wireless channels, owned by two POs, and $I = 10$ secondary users, 5 of which having $\alpha_l^i = 4 + \epsilon$, the other 5 having $\alpha_l^i = 4 - \epsilon$, with ϵ in the 0 to 0.1 range. In this way, since $\sum_{i \in \mathcal{U}} \alpha_l^i = 40 = constant$, the total flow on each wireless channel F_l is always the same for each ϵ value (see expression (10)). The other parameters, a_l, b_l and $\beta(l)$, are set as in the previous scenario.

Figure 2(a) reports the equilibrium flow (f_l^{i*}) transmitted over frequency spectrum F_l by two users belonging to each one of the two groups, as a function of ϵ. Furthermore, Figure 2(b) illustrates the ratio between the flow transmitted by users having $\alpha_l^i = 4 + \epsilon$ and those having $\alpha_l^i = 4 - \epsilon$.

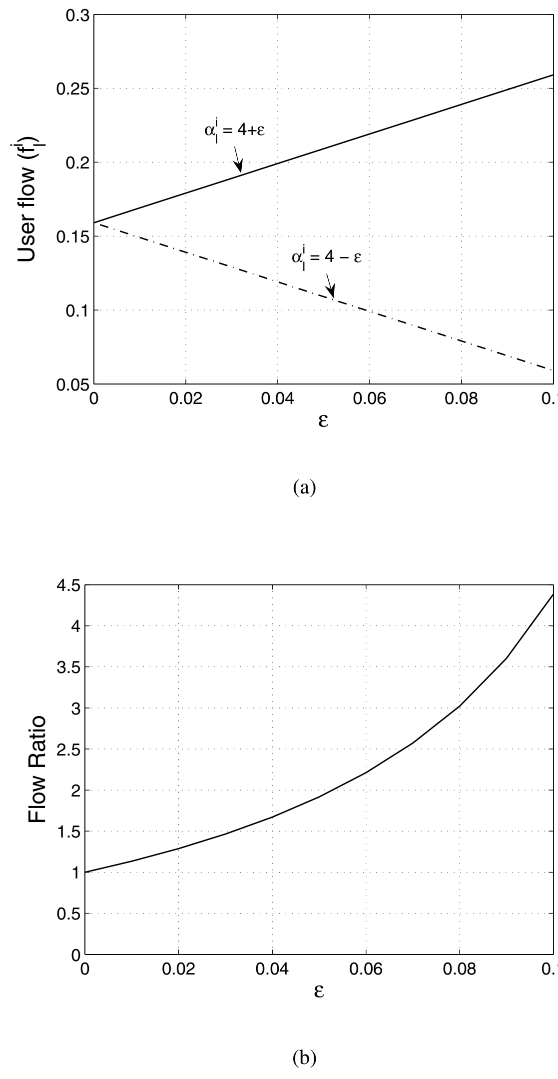

Fig. 2. CRN scenario with $I = 10$ secondary users, 5 of which having $\alpha_l^i = 4 + \epsilon$, the other 5 having $\alpha_l^i = 4 - \epsilon$: (a) equilibrium flow sent by each user on wireless channel F_l, and (b) ratio between the flow routed by secondary users having $\alpha_l^i = 4 + \epsilon$ and those having $\alpha_l^i = 4 - \epsilon$.

It can be observed that the amount of flow f_l^{i*} transmitted by each secondary user increases consistently with increasing α_l^i values, so that even small increases in ϵ lead to quite large differences in f_l^{i*}.

A variation of this scenario is then considered, where $I = 10$ secondary users have different per-flow utility values on the 2 wireless channels: 5 SUs have $\alpha_1^i = 3 + \epsilon$ on channel 1 and $\alpha_2^i = 4 + \epsilon$ on channel 2, the other 5 have $\alpha_1^i = 3 - \epsilon$ on channel 1 and $\alpha_2^i = 4 - \epsilon$ on channel 2. Figure 3 illustrates the equilibrium flows sent by each SU belonging to these two groups on each wireless channel, thus permitting to evaluate the flow variation due to the different utility levels perceived by such users on the two available wireless channels.

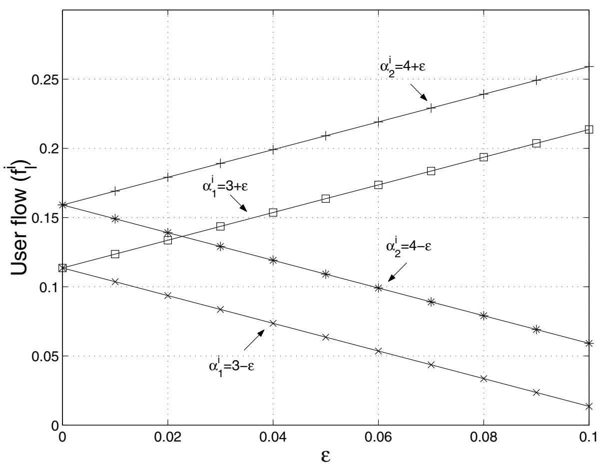

Fig. 3. Equilibrium flow sent by each user on $N = 2$ wireless channels with different per-flow utility values: 5 SUs have $\alpha_1^i = 3 + \epsilon$ on channel 1 and $\alpha_2^i = 4 + \epsilon$ on channel 2, the other 5 have $\alpha_1^i = 3 - \epsilon$ and $\alpha_2^i = 4 - \epsilon$ on channels 1 and 2, respectively.

B. Effect of the number of Secondary Users (I)

We further measured the revenue achieved by a PO as a function of the number of SUs, I, considering a CRN scenario with a single frequency spectrum, $\alpha_l^i = 4$, $a_l = 1$, $b_l = 0.5$ and $\beta(l) = 1$. Figure 4 illustrates the PO's revenue at the Nash Equilibrium point, $p_l^* f_l^*$, as a function of I. For simplicity, we consider the symmetrical users case studied in Section V. The equilibrium price, p_l^*, is independent of the secondary users' number, since it is equal to $p_l^* = \frac{\alpha_l - b_l}{2}$, while the equilibrium flow on wireless channel l, f_l^*, increases with increasing I values, $f_l^* = \frac{I(\alpha_l - b_l)}{2a_l(I+1)}$. Hence, the revenue achieved by the l-th Primary Operator increases with increasing I values, and is upper bounded by $\frac{(\alpha_l - b_l)^2}{4a_l}$ (which is approximately equal to 3.06 in this scenario).

VII. Conclusion

This paper studied the economic interaction between secondary users and primary network operators in Cognitive Radio Networks. Secondary users are characterized by elastic traffic, and face spectrum access costs. Primary Operators allocate the available spectrum setting appropriate prices to maximize their revenue. This problem was modeled with a two-stage (Stackelberg) game, where Primary Operators set prices per bandwidth unit and users respond by presenting a certain amount of flow to the network.

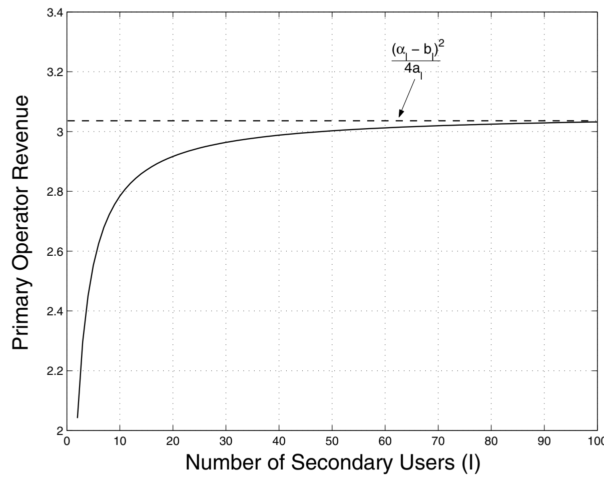

Fig. 4. Primary Operator revenue as a function of the number of users, I (Symmetrical secondary users case).

We proposed the utilization of polynomial pricing functions, which lead the system to unique and efficient Nash equilibrium points. Finally, we illustrated several numerical examples that provide insights into our proposed game.

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