Université Paris Descartes, BME-Paris Master - UE Math II - 2017-2018
Math test

Exercise 1 We consider the following function from $\mathbb{R}^2$ to $\mathbb{R}$:

$$f(x, y) = (x - y + 2)^2 + y^2 e^x.$$

1. Write the gradient of f, then find its critical points and show that f has a unique global minimizer.
2. Let E be the set $E = \{(x, y) \in \mathbb{R}^2, x + y = 1\}$. Show that E is a convex set.
3. If (x, y) is any point in $\mathbb{R}^2$, give the definition of the projection $\pi_C(x, y)$ of this point on a convex set C. Find the expression of $\pi_E(x, y)$.
4. Describe the projected gradient descent algorithm for minimizing the function f over the set E.

Exercise 2 Smith is in jail and has three dollars ; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. But the rules are unfair : if Smith bets m dollars, he wins m dollars with probability 0.4 and loses m dollars with probability 0.6. We consider two betting strategies :

a) the timid strategy : Smith bets one dollar each time,

b) the bold strategy : Smith bets all his money each time, unless he needs less to reach 8 dollars (for example if he has 6 dollars, he will bet only 2).

1. For each strategy a) and b), write the diagram of the Markov chain corresponding to the evolution of Smith's money. What are the transient and recurrent classes of these two chains ?
2. We consider the events $S_{n,i}$ ="Smith has i dollars at step n of the game" (for any i and n), and W ="Smith eventually reaches 8 dollars". Explain why the conditional probability $P(W|S_{n,i})$ does not depend on n. Therefore we can denote $p(i) = P(W|S_{n,i})$. Show that in each case a) and b) one can find linear equations between the $p(i)$. Explain how to deduce from this which is the best strategy for Smith between a) and b).