Université Paris Descartes, BME-Paris Master - UE Math II - 2015-2016
Math test

Exercise 1 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = x^4 - 2x^2 + y^2.$$

1. What are the critical points of f ?
2. Determine the type of each critical point : saddle point, local minimum or local maximum.
3. Using the equality $a^2 - 2a = (a - 1)^2 - 1$, prove that $f(x, y) \geqslant -1$. What are the global minimal points and maximal points of f ?

Exercise 2 Let X be a random variable with values in $\{1, \ldots, n\}$. We denote $p_i = P(X = i)$ for $1 \leqslant i \leqslant n$, and $\mathbf{p} = (p_1, \ldots, p_n) \in \mathbb{R}^n$.

1. Can $\mathbf{p}$ be any vector in $\mathbb{R}^n$? What are the constraints (equalities or inequalities on the coefficients) that it must satisfy ?
2. We consider C the set of all vectors $\mathbf{p}$ satisfying these constraints. Show that C is a convex set.
3. What is C in geometrical terms when $n = 1$, $n = 2$, $n = 3$?

Exercise 3 We consider the following weather model : if it rains two consecutive days, then it will rain on the next day with probability 2/3. If the weather is nice during two consecutive days, then the next day will be also nice with probability 3/4. Otherwise, if the current day is rainy and the previous day was nice, or the previous day was rainy but the current day is nice, then there is an even chance it will rain or be nice on the next day.

1. If the two first days are rainy, what is the probability that it will rain on the fourth day ?
2. Explain why this model cannot by defined as a Markov chain with two states, but that it can be defined as a Markov chain with four states. Write the diagram and transition matrix of this Markov chain.
3. If the two first days are nice, what is the probability it will rain on day 10 ? You just need to explain how to compute this probability, not to do the actual calculations.
4. Explain why the Markov chain is irreducible and aperiodic. Find its limit probability state.