Université Paris Descartes, BME-Paris Master - UE Math II - 2014-2015
Math test

Exercise 1

1. We throw a dice 1000 times in a row. Let N denote the number of times we obtain 6. What is the distribution of N ? What is its expectation ?
2. We change the game according to the following rule : we start by throwing one dice, and then at each step, if we obtained a 6 at the previous step, we throw 2 dices instead of one, otherwise we throw only one dice, and follow up with the same rule (obtaining a 6 then means that one of the two dices gave a 6).
   (a) Explain why this new game can be modelled as a Markov chain, write its diagram and transition matrix T.
   (b) What is the probability that we obtain a 6 at the third throw ?
   (c) What is the probability that we obtained a 6 at the third throw if we obtain a 6 at the fourth ?
   (d) Is the Markov chain irreducible ? Is it aperiodic ?
   (e) We assume that the limit probability state is a left eigenvector of T for the eigenvalue $\lambda = 1$. Compute this limit probability state.

Exercise 2 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x_1, x_2) = (x_1 - x_2)^2 + (x_1^2 + x_2^2 - 1)^2.$$

1. Compute the gradient of f and show that the only critical points are $(0,0)$, $(1/\sqrt{2}, 1/\sqrt{2})$ and $(-1/\sqrt{2}, -1/\sqrt{2})$.
2. Compute the hessian matrix of f, and for every critical point, decide, if possible from the hessian analysis, wether it is a local minimum, local maximum, or saddle point.
3. What is the global minimal point of f, or what are the global minimal points of f ?