**Université Paris Descartes**
Master 1 BME, UE Math Level 2.

Exercises on Ordinary Differential Equations

1. Let $a \in \mathbb{R}$. Show that $y(t) = \frac{1}{4}(t+a)^2$ is solution of the differential equation $x' = \sqrt{x}$. For which range of $t$ values this is true ? Find and draw the graphs of the maximal solutions of this equation. Do we have unicity of the IVP (initial value problems) for this equation ?

2. Among the following differential equations, specify which ones are linear and for these linear ODEs, specify which ones are homogeneous and/or with constant coefficients. In all cases, distinguish the ODEs which are in explicit form from those in implicit form, give the order of the equations and their dimensions. Also, when the equation is of order greater than 1, give an equivalent system of ODEs of order 1.

$$(a) : x' - \frac{2}{t+1}\, x = (t+1)^2 \qquad (b) : (t^2-1)\, x'' + t\, x' - x = 1$$
$$(c) : x' + t\, x = t^3 x^3 \qquad (d) : x'^2 + x' - t\, x = \cos t$$
$$(e) : x' + 4\, x = e^t \qquad (f) : x'' + \cos(t)\, x' + 2\, x = t$$
$$(g) : x''' + 4\, x^2 = 2 \qquad (h) : \begin{cases} x' = \sin(t)\, x + 3 \\ y' = x + 3y + e^t \end{cases}$$
$$(i) : \begin{cases} x' = x^2 + 3 \\ y' = 2\, x + z \\ z' = e^x \end{cases} \qquad (j) : \begin{cases} x'' = x - 5\cos(t)\, y' \\ y'' = 3\, y + e^t \end{cases}$$

3. *Linear equations of order 1 with constant coefficients* - Integrate (i.e. solve) the following differential equations :

a) $2x' = 2 - x$,
b) $x' + x = \cos t$,
c) $3x = 2x' - (1+t)e^t$,
d) $e^t x' - e^t x = 1$, with $x(0) = 1$.
e) $-x' - 2x = x'$, with $x(-1) = 0$.

4. *Numerical solutions of ODEs*
   a) Write a Matlab code to solve numerically the equation $x' = -x$, with initial condition $x(-5) = 1$, by performing 100 steps of the Euler scheme with stepsize $\delta t = 0.1$. Draw the graph of this numerical solution. On the same graph, draw also the true solution and compare.
   a) Now use the function `ode45` of Matlab to solve the same equation with same initial condition, over the range $-5 \leq t \leq 5$. Still on the same graph, draw this new numerical solution and comment.
   b) Use `ode45` to solve numerically the equation $x' = t\cos(x)$ for the initial condition $x(0) = 1$ and the range $0 \leq t \leq 10$.
   c) Still for equation $x' = t\cos(x)$, draw on the same graph the numerical solutions obtained with the different initial conditions $x(0) = 0$, $x(0) = 0.1$, $x(0) = 0.2$, ... $x(0) = 1$.