**Determinantal Point Processes (DPPs)**

- Model negative correlation thanks to the determinant of a kernel $K$: avoid bunching effect
- An exact sampling algorithm, based on the spectral decomposition of $K$
- Natural to apply DPPs to the space of patches, redundant and diverse

**Definition [1]**

Let $\mathcal{Y} = \{1, \ldots, N\}$ and $K$ be a hermitian $N \times N$ matrix, such that $0 \leq K \leq 1$, then the random subset $Y \subset \mathcal{Y}$, defined by

$$\forall A \subset \mathcal{Y}, \quad \mathbb{P}(A \subset Y) = \det(K_A),$$

is a DPP.

- Marginal probabilities of singletons: $\mathbb{P}(i \in Y) = K_{ii}$.
- Negative correlation between elements:
  $$\mathbb{P}\{i, j \in Y\} = \mathbb{P}(i \in Y)\mathbb{P}(j \in Y) - |K_{ij}|^2.$$

**The space of image patches and choosing an adapted kernel**

**Goal**: Sampling a representative subset $Y$ of $\mathcal{Y}$ the set of all the patches

Define the appropriate kernel $K$

$$K = L(L + I)^{-1}$$

with $L$ defined as

- Gaussian kernels [2] from
  - Distance of Intensity + Position,
  $$L_{ij} = \exp(-\|I_i - I_j\|^2 - \lambda \|\text{Pos}_i - \text{Pos}_j\|^2)$$
  - Distance in a PCA reduced space,
  $$L_{ij} = \exp(-\|\text{PCA}_i - \text{PCA}_j\|^2)$$
  - Quality/diversity kernel, where $q_i \in \mathbb{R}$, $\phi_i \in \mathbb{R}^D$,
  $$L_{ij} = q_i \phi_i^\top \phi_j$$

**Applications and further questions**

- Possible applications:
  - Image reconstruction or compression
  - Initialization of a k-means algorithm on patches
  
  *ex*: A. Coates and A. Y. Ng, Learning Feature Representations with K-means, Springer LNCs 7700, 2012

- Some studies suppose that patches are distributed as a Gaussian Mixture Model: Need to estimate the parameters.
  Problem: Too many redundant patches
  With DPP: Enlightened subsampling of the set of patches
  

- Need to study new patch similarity measures to improve the selection
  

**Results from basic image reconstruction**

**Comparison between kernels and the Poisson process**

**Figure**: Square root of the Mean Squared Error (MSE) between the original image of size $N$ and the reconstructed one in function of the number of patches selected

$$\text{MSE} = \frac{1}{N^2} \|u - u_K\|^2_2$$

**Figure**: Reconstructions from a Poisson point process and the DPP L2-PCA kernel from samples of 50 (two on the left) and 250 (on the right) patches

**Bibliography**
