Monte-Carlo Fork Search for Cooperative Path-Finding

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Outline

- Cooperative Path-Finding (CPF)
- Monte-Carlo Tree Search (MCTS)
- (Nested) Monte-Carlo Search (MCS)
- Monte-Carlo Fork Search (MCFS)
- Nested MCFS (NMCFS)
- CPF simulations
- Experimental results
- Conclusions
CPF example

- T=0
- Tnea=0

Each number corresponds to an agent.
• T=0
• Tnea=0

CPF example

Position

Goal

OK

Obstacle

Individual Path
CPF rules

• (On a graph)
• On a grid
  • 4-connectivity (or 8-connectivity)
  • obstacles
• Rules:
  • An agent can move on the neighbouring cell or stay
  • Rule 0: no two agents can be on the same cell
  • Rule 1: no two agents can swap
  • Circularity is possible
• T=0
• nea=0

CPF example

Position
Goal
OK

Obstacle

Elementary Action
CPF example

- T=1
- Tnea=9
- nea=5

Position
Goal
OK

Elementary Action
CPF example

- $T=2$
- $T_{nea}=14$
- $nea=2$

Position

Goal

OK

Elementary Action
### CPF example

- **T=3**
- **Tnea=16**
- **nea=4**

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- **Position**
- **Goal**
- **OK**

**Obstacle**

**Elementary Action**
CPF example

- $T=4$
- $T_{nea}=20$
- $nea=6$

Position

Goal

OK

Elementary Action

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</table>

Obstacle

The CPF example shows a grid with positions, obstacles, goals, and OK areas. The numbers indicate the position and movement within the CPF framework.
CPF example

- $T=5$
- $T_{nea}=26$
- $nea=6$
CPF example

- T=6
- Tnea=32
- nea=4
 CPF example

- T=7
- Tne=36
- nea=4

Position

Goal

OK

Elementary Action
 CPF example

- $T=8$
- $T_{nea}=40$
- $nea=6$

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</table>

Elementary Action

Position

Goal

OK
CPF example

- $T=9$
- $T_{nea}=46$
- $nea=5$

Elementary Action
 CPF example

- \(T=10\)
- \(T_{nea}=51\)
- \(nea=1\)
CPF example: end!

- $T=11$
- $T_{nea}=52$

![ CPF Diagram ]
CPF optimization target

- **Sequentiality**
  - sum of individual costs
  - i.e. *Total number of elementary actions* ($T_{nea}$)
  - Most of work in the literature

- **Simultaneity**
  - *Global elapsed time* ($T$) = *Number of timesteps*
  - Our work + TOMPP (Yu 2012)
Complexity

- **Number of joint actions is exponential in the number of agents**
  - 10 agents and 5 actions yield $5^{10} \approx 10^7$ joint actions

- « Best first » searches (MCTS, A*) do not work
  (even on the simple example)

- **Challenging!**
CPF previous work

- WHCA* (Silver 2006)
- A* + Operator Decomposition (OD) (Standley 2010)
- A* + Independent Detection (ID) (Standley 2011)
- Incremental Cost Tree Search (ICTS) (Sharon 2011)
- Multi-Agent Path Planning (MAPP) (Botea 2011)
- Push & Swap (Luna 2011)
- TASS (Khorshid 2011)
- Time-Optimal Multi-Agent Path Planning (TOMPP) (Yu 2012)
- CPF video game benchmarks (Sturtevant 2012)
Overall approach

- MCTreeSearch
  - Nested MCTS
    - N-player games

- MCSearch
  - Nested MCS
    - 1-player game

- MCForkSearch
  - Nested MCFS
    - CPF
Why Monte-Carlo?

- Evaluating a non terminal position:
  - a lot of evaluations of terminal positions reached with simulations
  - a knowledge-based evaluation function
Why Monte-Carlo Tree Search?

- Choosing the next action to perform must be accurate.
- 2-player games, n-player games, 1-player game (planning).
Sequential decision making

- The first action is performed...
- … another MCTS is launched to choose the next action...
Sequential decision making

- Then the second one is performed...
Sequential decision making

- And so on...
Sequential decision making

- Until the sequence reaches the end of the domain.
Level one simulation

- Level 1 simulation – or « smart » simulation -
- … usable within another MCTS
Nested MCTS

- **Level N MCTS** performs a simulation by using level N-1 simulations

- Level 0 simulations:
  - Random simulations
  - Pseudo-random simulations

- MCTS is too costly.

- Is it possible to design an algorithm that could be nested several times?
(Nested) MCS

- Depth-one greedy search using one simulation per child node (Cazenave 2009)

- The CPU time is less important

- The levels can be nested several times (2 or 3 or 4 depending on the domain)

- Planning, 1-player games, expression discovery, weak schur numbers, morpion solitaire, etc.
CPF Complexity

- Number of joint actions is \textit{exponential in the number of agents}
  - 10 agents and 5 actions yield \(5^{10} \approx 10^7\) joint actions

- «Best first» searches (MCTS, MCS, A*) do not work on the simple example
  - ... because of the branching factor
How to adapt the MCS approach?

- **Question 1**: How to choose a move at random and perform only one simulation?

  ? ? ?

- **Question 2**: How to develop a search without being stuck near the initial node?

  ? ? ?
MC Fork Search rationale
MC Fork Search rationale
MC Fork Search rationale
MC Fork Search rationale

- fork the next simulation along the current best sequence

- **Use UCB** (or any other rule) on the whole tree
- No top down order
int NMCFS(start, goal, bestSeq, lev) {
    if (lev == 0) then return sample(start, goal, bestSeq)
    n = 1; lmin = +\infty; actualSeq; Node root(a)
    while (n \leq it) do {
        Node nd = root.selectNode(); pos = nd.position
        l = NMCFS(pos, b, actualSeq, lev-1)
        If (l + nd.depth < lmin) then {
            lmin = l + nd.depth
            bestSeq = seq(root(a), nd) + actualSeq
        }
        nd.backUp(l); nd.append(actualSeq, l, b); n = n + 1
    }
    return lmin
}
UCB rule adapted to MCFS

\[
\text{argmin}_{\text{builtTree}} \left( \text{lmin+depth} - C \left( \text{variance log(n)/(1+nforks)} \right)^{1/2} \right)
\]

- Selection is global
- Exploitation focused on nodes belonging to the shortest sequences
- Exploration focused on nodes with high variance and low number of forks
CPF simulations (1/2)

- Optimal individual paths are pre-computed (Botea 2011)

- Each agent knows its optimal elementary actions.
  - (1) Greedy choice
  - (2) Pseudo-random choice
- Each agent expresses a « wish » (the next cell)

- Collision management
  - Choose the agent that gives the way at random

- Iterative process to choose one joint action
CPF simulations (2/2)

- Choice of the **joint action**: 

  While the joint action is not correct {
    Ask each agent for its wish
    Manage the collisions
  }

- **Progressive relaxation**:
  - (1) Optimal actions only
  - (2) Optimal actions + stay
  - (3) All actions (random walk)
Experiments

- **Set of problems**
  - (Khorshid 2011) \( \rightarrow \) Tnea
  - (Luna & Bekris 2011) \( \rightarrow \) Tnea
  - CPF N-puzzles (Yu 2012) \( \rightarrow \) Time
  - Many-agents-large-grid problems \( \rightarrow \) Tnea

- 3.2 Ghz CPU with 6 Gbytes memory

- **Comparing results?**
  - Our work: optimizing 'Time'; (however 'Tnea' available)
  - Most of other work: optimizing 'Tnea'
Reference programs

- **TASS** (Khorshid 2011)
  - \#agents = gridsize – 4
  - knowledge-based, fast, complete, sub-optimal.
- **Push & Swap** (Luna & Bekris 2011)
  - \#agents = gridsize – 2
  - knowledge-based, fast, complete, sub-optimal.
- **TOMPP** (Yu & la valle 2012)
  - Network flow analogy, uses linear programming
  - Time-optimal.
Results on Khorshid's problems

Comparison with Tree-based Agent Swap Strategy (TASS)

<table>
<thead>
<tr>
<th>T</th>
<th>Tnea</th>
<th>time</th>
<th>lev</th>
<th>it</th>
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<td>≤15</td>
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<td>516</td>
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<td>≤18</td>
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## Results on Luna & Bekris problems

### Comparison with Push & Swap

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<td>12h</td>
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Results on N-puzzles with no hole

- N=8, 15, 24: one hole; N=9, 16, 25: no hole.

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<tr>
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<th>Branch. factor</th>
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<td>$\approx 10^5$</td>
<td>7</td>
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<td>$\approx 3\times10^4$</td>
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Result on a 25-puzzle (from Yu 2012)

- $T=0$
- $T_{nea}=0$

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**Result on a 25-puzzle** (from Yu 2012)

- $T=0$
- $T_{nea}=0$

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Individual paths
A joint action = several cycles

- T=0
- Tnea=0
- nea=24
### Result on a 25-puzzle

- **T=1**
- **Tnea=24**
- **nea=10**

<table>
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### Elementary Action

- 1
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- 11
- 17
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- 21
- 7
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- 24
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- 22
- 23
- 24
- 25
Result on a 25-puzzle

- $T=2$
- $T_{nea}=34$
- $nea=10$

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
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<td>21</td>
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<tr>
<td>16</td>
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<td>25</td>
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</tbody>
</table>

Elementary Action
### Result on a 25-puzzle

- T = 3
- T_{nea} = 44
- n_{ea} = 14

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<table>
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<th>Goal</th>
<th>OK</th>
</tr>
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Result on a 25-puzzle

- $T=4$
- $T_{nea}=58$
- $nea=22$

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Elementary Action
### Result on a 25-puzzle

- **T=5**
- **Tnea=80**
- **nea=12**

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**Position**

**Goal**

**OK**

**Elementary Action**
Result on a 25-puzzle

- $T=6$
- $T_{nea}=92$
- $nea=20$

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Elementary Action
Result on a 25-puzzle

- $T = 7$
- $T_{nea} = 112$
- $nea = 8$

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Elementary Action
**Result on a 25-puzzle**

- \(T=8\)
- \(T\text{nea}=120\)
- \(\text{nea}=0\)

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**Elementary Action**

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6 7 8 9 10
11 12 13 14 15
11 12 13 14 15
16 17 18 19 20
16 17 18 19 20
21 22 23 24 25
21 22 23 24 25
```
Many-agents-large-grid result

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Many-agents-large-grid limits

- Main limit:
  - computation and storage of distances between cells

- For 200x200 grids:
  - Memory size in $200^4 = 1.6 \times 10^9$
  - Pre-computation of distances between cells
  - Video game benchmarks: 500x500 grids (Sturtevant 2012)

- Other limit: Time for running one simulation
Algorithm features

- **Memory use**
  - No problem with the nested version

- **Completeness**
  - Worst case: the meta-graph is browsed by a meta-random walk
  - Solvable problem = meta-graph connected = random walk reaches all states

- **Anytime**

- **Near-optimality**
  - $h = \max_{\text{agent}} \text{pathLength}(\text{agent})$
  - No other measure of distance to optimality
Conclusions and perspectives

- Nested MCFS solves CPF problems
  - Not solved by classical solvers (A*, MCTS, MCS)
  - N-puzzles without hole, Khorshid, Luna&Bekris, Many-agents
- MCFS is not sensitive to the branching factor
- Features: anytime, complete, near-optimal
- Future work
  - How to set #level and #fork automatically ?
  - Test on other very-high branching factor problems
  - CPF: 8-connectivity, larger grids
  - Diameter of the graph of the nxn-puzzle
  - 2-player games ?
Thank you for your attention!

- Questions?

- brunobouzy@parisdescartes.fr